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BALLISTIC LIMIT REGRESSION ANALYSIS FOR SPACE STATION FREEDOM METEOROID AND SPACE DEBRIS PROTECTION SYSTEM

Final Report

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ρ_1	Density of the Bumper Material
ρ_2	Density of the Rear Wall
ρ_{wp}	Density of the Witness Plates
v	Velocity
t_1	Bumper Thickness
t_2	Rear Wall Thickness
t_{wp}	Witness Plate Thickness
θ	Angle between Target Normal and Projectile Path (Obliquity)
x	MLI Position, Distance from Rear Wall
d	Projectile Diameter
n_{wp}	Number of Witness Plates Penetrated
h	Maximum Crater Depth on Rear Wall
P	Total Areal Density Penetrated
P^*	Penetration Parameter
P_c	Critical value of Penetration Parameter
c_i	i^{th} Regression Coefficient

ACRONYMS

ANOVA	Analysis of Variations
BLC	Ballistic Limit Curve
HITS	Hypervelocity Impact Testing Summary
LEO	Low Earth Orbit
MLI	Multi-Layered Insulation
PNCF	Probability of No Critical Flaw
PNP	Probability of No Penetration
SSF	Space Station Freedom

1.0 SUMMARY

Relationships defining the ballistic limit of Space Station Freedom's (SSF) dual wall protection systems have been determined. These functions were regressed from empirical data found in Marshall Space Flight Center's (MSFC) Hypervelocity Impact Testing Summary (HITS) for the velocity range between three and seven kilometers per second. A stepwise linear least squares regression was used to determine the coefficients of several expressions that define a ballistic limit surface. Using statistical significance indicators and graphical comparisons to other limit curves, a final set of expressions is recommended for potential use in Probability of No Critical Flaw (PNCF) calculations for Space Station. The three equations listed below represent the mean curves for normal, 45 degree, and 65 degree obliquit y ballistic limits, respectively, for a dual wall protection system consisting of a thin 6061-T6 aluminum bumper spaced 4.0" from a .125" thick 2219-T87 rear wall with multiple layer thermal insulation installed between the two walls.

Normal obliquity:

$$d_c = 1.0514\ v^{0.2983}\ t_1^{0.5228}$$

45 degree obliquity:

$$d_c = 0.8591\ v^{0.0428}\ t_1^{0.2063}$$

65 degree obliquity:

$$d_c = 0.2824\ v^{0.1986}\ t_1^{-0.3874}$$

Plots of these curves are provided in section 5.0 of this report.

A sensitivity study on the effects of using these new equations in the probability of no critical flaw analysis has indicated a negligible increase in the performance of the dual wall protection system for SSF over the current baseline. The magnitude of the increase was 0.17% over 25 years on the MB-7 configuration run with the Bumper II program code.

2.0 INTRODUCTION

2.1 PURPOSE

Hypervelocity impact testing has been performed in the Light Gas Gun Facility at MSFC since 1985. This testing has been directed toward the development of a meteoroid and space debris protection system design for SSF. The information gathered from this testing has been formally recorded in a Lotus database entitled Hypervelocity Impact Testing Summary (HITS).

The purpose of this analysis is to determine the ballistic limit of dual wall meteoroid and space debris protection system, similar to the proposed system for SSF, using HITS data. The empirical relationships derived are intended for use in the design and verification of the SSF protection system.

Two methods are used to determine the empirical ballistic limit curves and, an Analysis of Variations (ANOVA) is performed to indicate the statistical significance of these curves. In order to quantify the scatter in the test data, confidence intervals are determined for each regression.

2.2 BACKGROUND

Meteoroid and space debris impacts are anticipated to occur on the exterior of the Space Station during its service life in a low earth orbit (LEO). As a result, the external walls are required to be designed to minimize the risks associated with these impacts. The SSF requirement document [8] states that the probability of an anticipated impact to cause failure of the pressure wall will be less than 0.45% over a ten year period. In order to calculate this probability, ballistic limits must be determined.

The definition of a ballistic limit varies depending on the method of analysis being employed. For this analysis, the ballistic limit is defined as the velocity at which a specified projectile will just barely penetrate the second wall (or rear wall) of a dual wall structure. Failure of the second wall by cracking or spalling is considered penetration since pressure loss would occur under those circumstances.

2.2.1 Ballistic Limit

The ballistic limit for dual wall structures is governed by processes whose phenomenologies change as the impact velocity increases. Specifically, the ballistic limits can be subdivided into three velocity regimes: ordinance, shatter, and hypervelocity. These regimes are differentiated by the relative strengths of the projectile and target for given impact pressures. The velocity range considered for this analysis is the shatter regime and, for aluminum spheres impacting aluminum targets, that regime is roughly between two and eight kilometers per second (km/sec). In this velocity range, the mechanics of penetration changes from impacts at lower velocities where projectiles remain intact throughout the penetration event, to impacts at higher velocities where the projectile becomes completely pulverized during penetration of the first wall or bumper, as it will be referred to in this report. This section of the ballistic limit curve is highly nonlinear due to the randomness of the shatter mechanisms causing the projectile to breakup. However, Burch indicated in [1] that the general shape of the ballistic limit curve, in this velocity range for this target configuration and normal obliquity, is monotonicly increasing with velocity which indicates a reduction in damage (or penetration) as velocity increases.

2.2.2 Application

A computer code, known as "Bumper", uses ballistic limit curves (BLC) and an estimation of the anticipated environment[1] to determine the PNCF for spacecraft structures. PNCF is a statistical

[1] Space Station Freedom program recognizes the environment specified in [6].

measure of the penetration resistance of a spacecraft's protection system.

For each of the three velocity regimes, "Bumper" uses a different BLC. In the shatter regime, the program allows the choice of several BLCs. One of these choices, denoted "Boeing Interp", accesses a look-up table of data points and with a linear interpolation routine determines a critical diameter for various bumper thicknesses over a range of impact obliquities. The look-up table of points that lie on the BLC is generated from the test data; therefore, the regression equations must be applicable over bumper thicknesses between 0.040" and 0.080" and obliquities[2] up to 65° for the SSF dual wall.

[2] Obliquity is the angle between the projectile velocity vector and the outward normal of the target.

3.0 TEST AND DATA DESCRIPTION

All data considered in this analysis was generated in testing performed in the Light Gas Gun Facility at Marshall Space Flight Center. Since this analysis and desired ballistic limits are specific to Space Station, only shots made against targets similar to its proposed dual wall configuration were considered. This reduces the required complexity of the ballistic limit expressions and, in theory, should increase the accuracy of the regression. The following discussion provides more specific information about the tests used to generate the ballistic limit curves.

3.1 PROJECTILE CONFIGURATION

The only projectile type considered for this analysis was a pure aluminum sphere. 1100-O (pure annealed aluminum) was used extensively in testing because its average density is very near the estimated average density of space debris as specified in [6]. Since only one material is considered in this analysis, spherical diameter and projectile mass are directly related and diameter can be used to convey ballistic limit information. In this report, a critical projectile diameter is plotted as a function of impact velocity to portray a ballistic limit against a specific target.

3.2 TARGET CONFIGURATION

Figure 1 shows a dual wall target configuration composed of two walls spaced 4.0" apart with a Multi-Layered Insulation (MLI) blanket located between the walls. The bumper is 6061–T6 aluminum sheet that ranges in thickness between .032" and .080". The rear wall is 0.125" thick 2219-T87 aluminum sheet. The actual pressure wall of SSF is proposed to be waffle plate; however, it is 0.125" thick between the ribs and would be expected to behave similar to plain sheet stock for penetrations near the ballistic limit. The target is usually backed up by three 0.020" 7075 aluminum witness plates; however, more plates are often used for high momentum

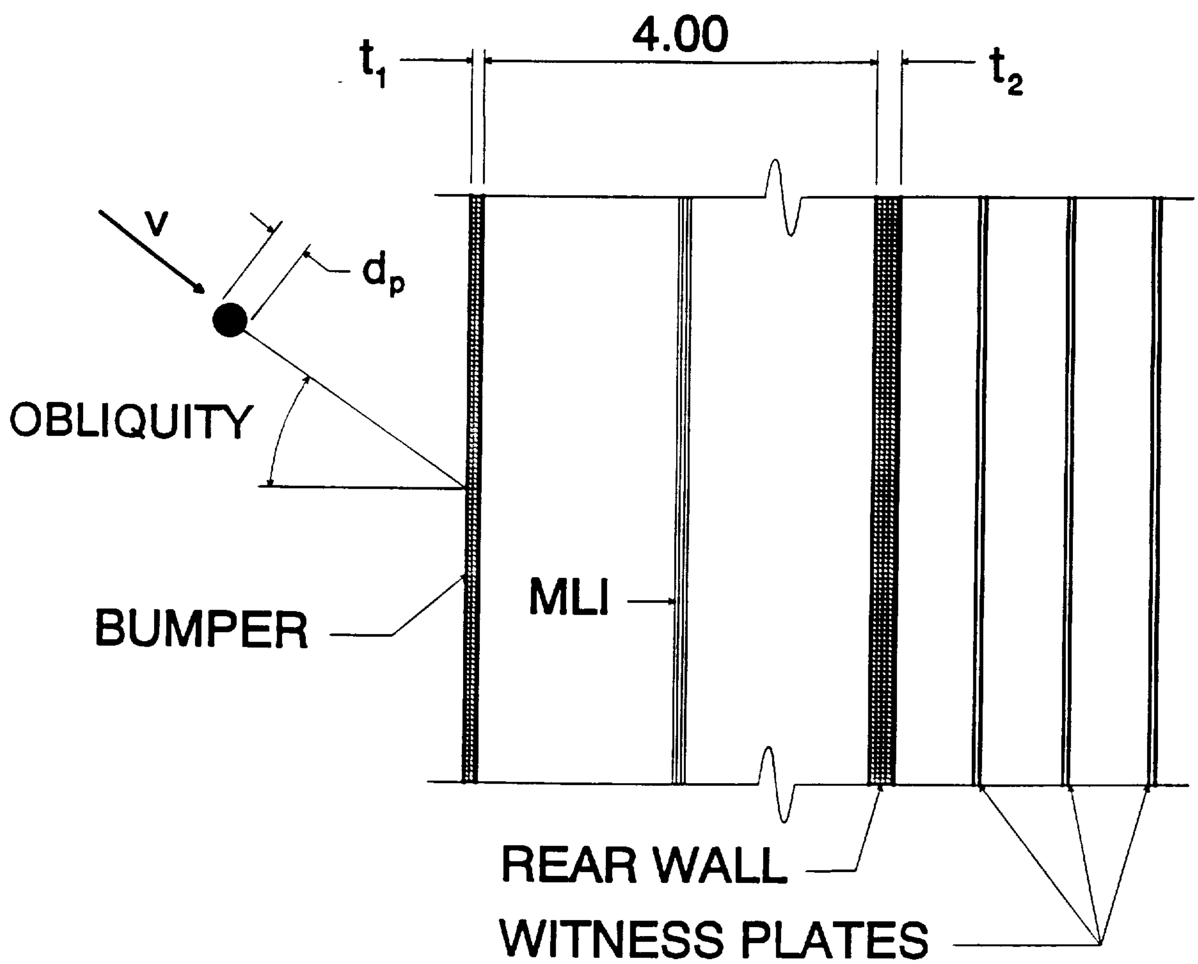

Figure 1. Dual Wall Target Configuration

shots.

The bumper thickness varies depending upon the specific requirements for the particular SSF component. In fact, this is the predominant parameter of variance to be considered in design optimization of the protection system. Therefore, bumper thickness will be handled in the regression analysis as an independent variable and the BLCs will be applicable over the range of bumper thicknesses indicated.

3.3 DATA SUMMARY

The HITS database was searched for tests on dual wall targets with 6061-T6 bumpers and 0.125" 2219-T87 rear walls spaced 4" apart, impacted with 1100-O pure aluminum spheres at any available obliquity, MLI position, and bumper thickness. In addition to the geometric search parameters, other search parameters included base line requirements on the information available for each shot. For instance, shots that penetrated the rear wall must have witness plate damage information and shots that did not penetrate the rear wall must have crater depth information. If a test record indicated multiple holes in the bumper, then the projectile was assumed to have broken up before impacting the target. This was found to be the case in four tests and the shots were removed from the regression dataset.

A total of 385 hypervelocity impact tests, fired at velocities between two and eight km/sec, were found to comply with these search parameters. This data along with a list of discarded shots are provided in Appendix A.

3.3.1 Shot Summary of the Regression Dataset

Tables 1 through 6 summarize shot diversity for the 385 shots used in this analysis. The majority of the data is for targets where MLI was placed near the bumper or against the rear wall. In the actual SSF configuration the MLI is centered between the walls. Nineteen shots, applicable to this regression, have been made against targets with MLI centered between the walls, but all of them were fired at normal obliquity on 0.063" bumpers. The 221 shots used in the final analysis are indicated by the asterisks.

Figure 2 is a sample plot of some of the shot results indicating the final condition of the rear wall.

Table 1. Shot Occurrences with No MLI Present

Bumper Thickness (in.)	Diameter (in.)	Number of Shots (above/below 4.75 km/sec)						
		Obliquity						
		0°	30°	45°	55°	60°	65°	75°
.080	.375					0/1		0/1
	.350	0/1		1/1		1/1		
	.313			0/1				
	.300	0/1						
	.250			0/1				
	.187	5/1						
.063	.313	0/3	0/1	0/2		0/1	0/1	0/3
	.300	0/2						
	.262	0/2						
	.250	2/9	0/6	0/4	0/3		2/3	
	.187	3/0	0/2	2/6		0/2	2/0	
	.125	1/0						
.040	.250	3					4	
	.187			8				
	.125			3				
.032	.300			1				

Table 2. Shot Occurrences with MLI near the Bumper

Bumper Thickness (in.)	Diameter (in.)	Number of Shots (above/below 4.75 km/sec)						
		Obliquity						
		0°	30°	45°	55°	60°	65°	75°
.080	.313	3*						
	.250	4*						
.063	.375	0/1*						

* Shots used in final regression analysis.

Table 3. Shot Occurrences with MLI at 3.75" from the Rear Wall

Bumper Thickness (in.)	Diameter (in.)	Number of Shots (above/below 4.75 km/sec)						
		Obliquity						
		0°	30°	45°	55°	60°	65°	75°
.080	.313	0/1*		6/6*		2/2		5/6
	.250	0/1*		2/6*		2/2		2/2
	.187			2/5*		2/2		2/2
.063	.313	0/5*						0/1
	.250	2/6*		0/3*				
	.187	3/0*		0/2*				
.050	.313			1/0*		3/2		3/1
	.250			2/10*		2/2		2/2
	.187			2/2*		4/2		2/2

* Shots used in final regression analysis.

Table 4. Shot Occurrences with MLI at 0.90" from the Rear Wall

Bumper Thickness (in.)	Diameter (in.)	Number of Shots (above/below 4.75 km/sec)						
		Obliquity						
		0°	30°	45°	55°	60°	65°	75°
.063	.375	0/1*						

* Shots used in final regression analysis.

Table 5. Shot Occurrences with MLI Centered between Walls

Bumper Thickness (in.)	Diameter (in.)	Number of Shots (above/below 4.75 km/sec)						
		Obliquity						
		0°	30°	45°	55°	60°	65°	75°
.063	.375	0/4*						
	.313	0/8*						
	.250	4/3*						

* Shots used in final regression analysis.

Table 6. Shot Occurrences with MLI on the Rear Wall

Bumper Thickness (in.)	Diameter (in.)	Number of Shots (above/below 4.75 km/sec)						
		Obliquity						
		0°	30°	45°	55°	60°	65°	75°
.080	.313	3*		4*			1*	
	.300	4*						
	.250			1*			2*	
	.187			1*				
.063	.375	1*						
	.350			2*			2*	
	.313	5*	1	5*				
	.300	3*		1*			3*	
	.262	1*						
	.250	4*		5*			3*	
	.187	5*	1	2*				
.040	.375			1*				
	.350						2*	
	.313	2*		5*			6*	
	.300						5*	
	.250	5*	1	7*			3*	
	.187	5*	3	6*				
.032	.313			1*			3*	
	.250	1*		3*			4*	
	.187	4*		3*			2*	

* Shots used in final regression analysis.

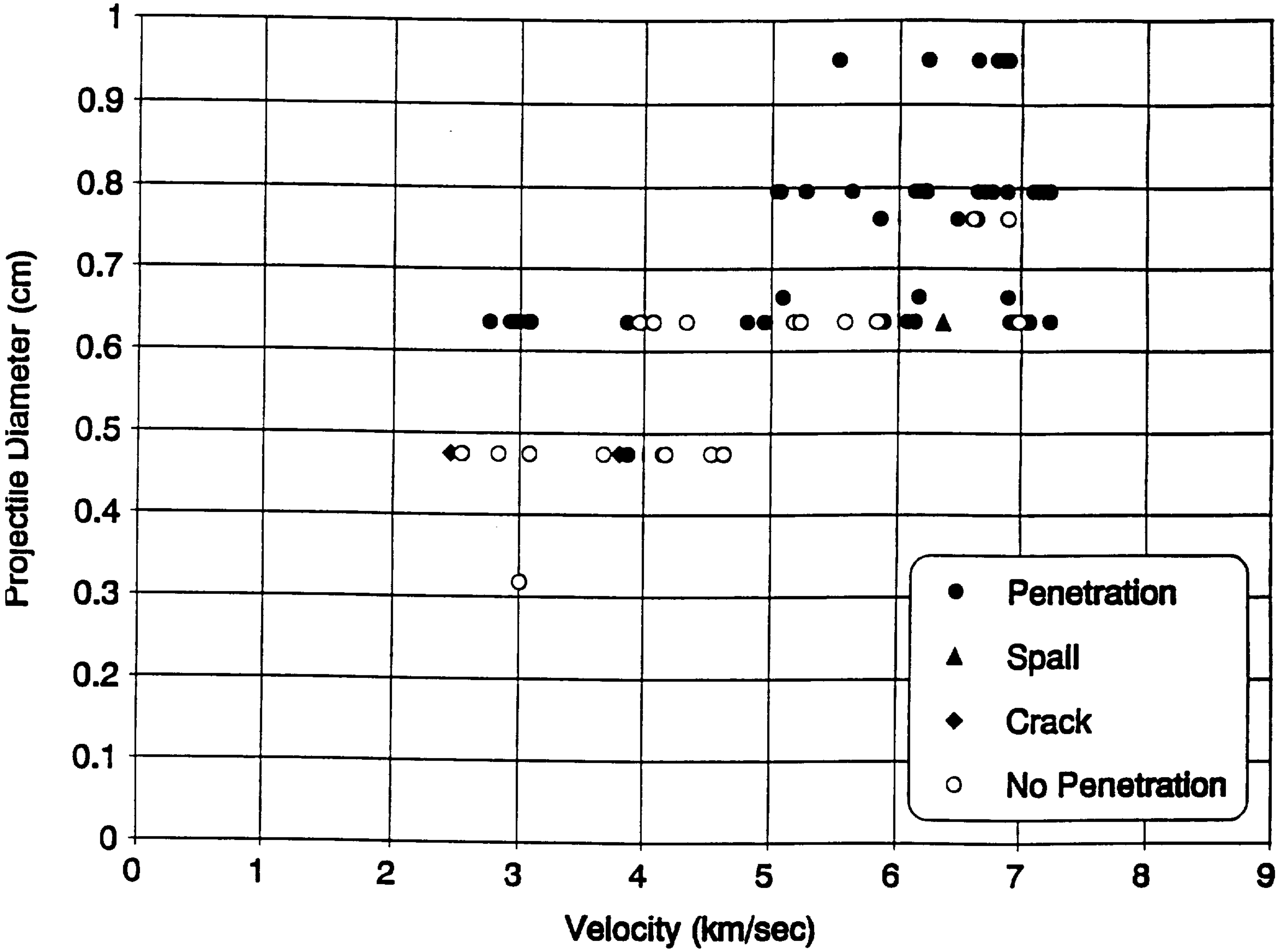

Figure 2. Raw Data Plot for 0.063" Bumper Impacted Normally by a 0.250" Projectile.

4.0 ANALYSIS DESCRIPTION

Two linear regression methods were used to derive expressions from the available data in the HITS database. The following sections provide detailed descriptions of the methods used in this analysis to generate those expressions. Since there were so many regressions performed, a single ANOVA was performed to determine the level of confidence for the final set of curves generated.

4.1 PENETRATION PARAMETER

No matter which regression method is used, a dependent penetration parameter is required to provide a dependent variable that relates the penetration process to the independent test variables. The penetration parameter (P) is a calculated variable that characterizes the amount of damage sustained by the target.

For this analysis, the penetration parameter is defined as, "the total areal density penetrated plus one". The areal density is incremented by one so that the natural logarithm does not go to negative infinity when the bumper completely defeats the projectile (i.e., when $P=0$). The necessity for taking the logarithm will become apparent in section 4.2.2. The Penetration Parameter may be written as:

$$P^{*} = P + 1 \tag{1}$$

The total areal density is defined as a step function with respect to rear wall penetration. For shots that did not penetrate the rear wall, the total areal density is the product of the depth of the deepest crater found on the wall and the density of the rear wall (2.851 gm/cc for 2219-T87 aluminum). Equation (2) represents this quantity. For shots where penetration of the rear wall

$$P = h\,\rho_2 \tag{2}$$

did occur, the number of witness plates penetrated indicates the amount of damage. It was assumed that, if a witness plate was penetrated, then half of the next witness plate was also penetrated. Therefore, the penetration parameter becomes the areal density of the rear wall plus the areal density of the number of witness plates penetrated plus one half. This may be written as:

$$P = t_2 \rho_2 + \left(n_{wp} + \frac{1}{2}\right) \rho_{wp} t_{wp} \tag{3}$$

Critical penetration corresponds to the value of the penetration parameter equal to the areal density of the rear wall. When this occurs, the rear wall should, theoretically, be "just" penetrated. The following equations define this parameter and the numerical values given correspond to the SSF dual wall target configuration.

$$P_c = \rho_2 t_2 = 0.3175cm \left(2.851 \frac{gm}{cm^3}\right) = 0.9052 \frac{gm}{cm^2} \tag{4}$$

$$P_c^* = P_c + 1 = 1.9052 \tag{5}$$

Figure 3 is a plot of the penetration parameter versus impact velocity for the largest group of data. Notice that the shots below critical penetration ($P_c^* = 1.9052$) are randomly dispersed and, conversely, the shots above critical occur in discrete groupings, coinciding with the number of witness plates penetrated. Including tests for both penetrated and non-penetrated targets in the analysis, provides continuity in the penetration descriptor and should lead to better regression fits as long as the tests were made near the ballistic limit.

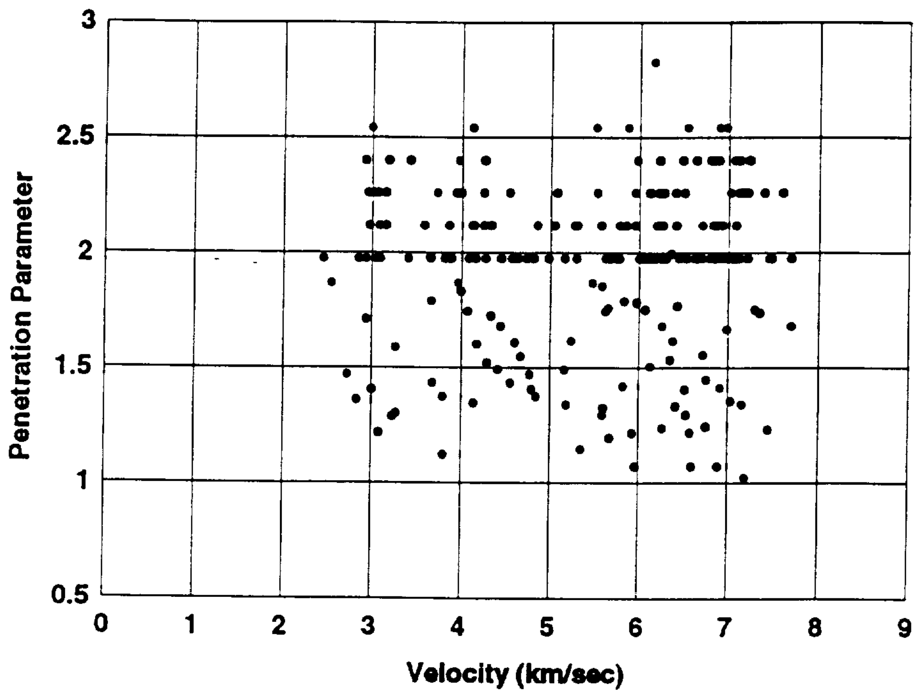

Figure 3. Penetration Parameter versus Velocity - Representative Sample.

4.2 REGRESSION ANALYSIS TECHNIQUES

Two regression techniques were investigated in this analysis, a Grouped Linear analysis and, the more familiar Least Squares Multiple Regression.

4.2.1 Grouped Linear Regression

This method employs a grouping technique followed by a linear regression constructed to force the line through a known point based upon a "single wall" ballistic limit equation. The term "Grouped Linear" has no historical basis and is used descriptively to indicate the following procedure.

The shot information (e.g., shot number, penetration parameter, and projectile diameter) for a specific bumper thickness, Multi-Layer Insulation (MLI) position, and obliquity is separated into groups based on velocity. With an acceptable spacing of velocity (set at ± 0.5 km/sec for this analysis), the only remaining variables are Projectile Diameter and Penetration Parameter. Figure 4 graphically illustrates one set of grouped shots. A linear regression of this data would represent the functional relationship between the diameter of a projectile and the damage it would cause for a given impact velocity, obliquity, and target configuration. The results of this regression should reflect the physical phenomena governing the event. However, because of the excessive scatter expected from highly non-linear phenomena and the small amount of available data, significance of the curves would be highly questionable. To resolve this problem, the line can be anchored at one end by recognizing that the ballistic limit of the bumper occurs when P^* equals one. Therefore, a single wall penetration equation[3] used to anchor the regression at a known point (e.g., the ballistic limit of the bumper) would reduce the effects of the scatter and small data samples. A linear regression is then performed to position a line passing through the point representing the ballistic limit of the bumper and the centroid of the data. As illustrated in Figure 4, the diameter indicated by the linear function when $P^* = P_c^* = 1.9052$, provides a critical projectile diameter (d_c) at the grouped velocity. This process is repeated for all groups of data across the velocity range.

With a critical projectile diameter for each velocity, a continuous BLC can be generated by fitting a weighted curve to the data. The weighing should be based on the number of points contained in each group.

[3] The Fish-Summer single wall penetration equation was used in this analysis due to its correlation with test data presented in [2].

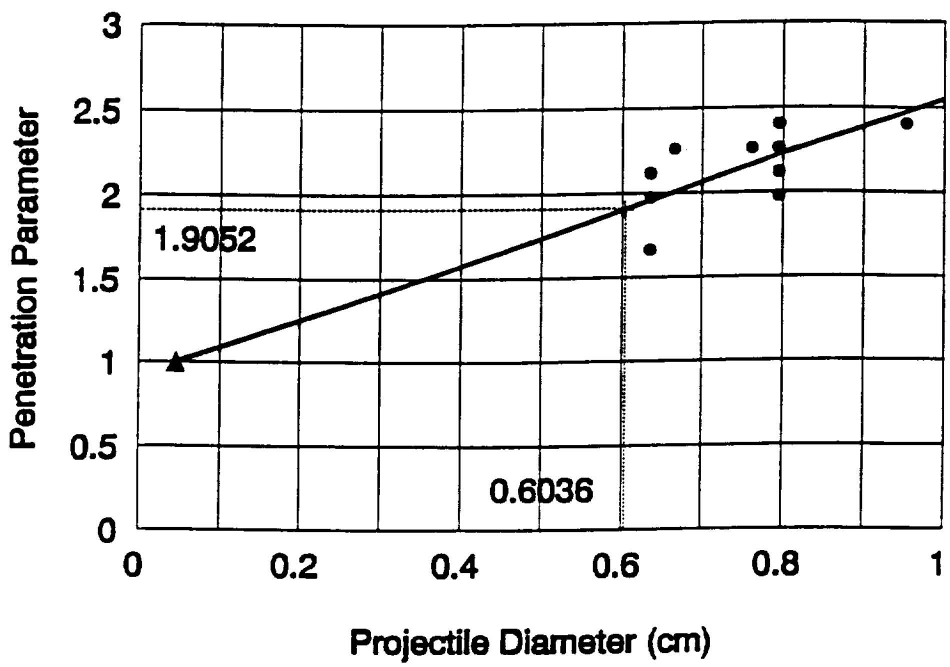

Figure 4. Dual Wall Target Test Data Grouped by Velocity for Constant Obliquity and Bumper Thickness

4.2.2 Multiple Regression

Multiple Regression refers to a multivariate linear least squares regression of a non-linear equation mapped into linear space. In this analysis, mapping was performed by imposing algebraic laws of logarithms on a posynomial form and expanding.

Assume a general posynomial form such as:

$$P^{*} = e^{c_1} v^{c_2} t_1^{c_3} (\cos\theta)^{c_4} d^{c_5} \tag{6}$$

where,

P^{*}	is Penetration Parameter
e	is the exponential function
v	is the Impact Velocity
t_1	is the Bumper Thickness
θ	is the Obliquity of the Projectile's Trajectory
d	is the Projectile Diameter
c_i	is the i^{th} Regression Coefficient.

Then, map the form into linear space by taking the natural logarithm and expanding to get the polynomial expression shown below:

$$\ln(P^{*}) = c_1 + c_2 \ln(v) + c_3 \ln(t_1) + c_4 \ln(\cos\theta) + c_6 \ln(d) \tag{7}$$

Apply linear least squares regression techniques to determine the coefficients. This method is outlined in [4] and is similar to the method used by Burch to generate the widely accepted work presented in [1]. Also, Dr. Robert Mog used this method in his work on posynomial regression analysis [7].

The primary limitation of this method, or any method of regression, is the correctness of the assumption of the model form. The posynomial form is assumed in this analysis; therefore, the relationships between the dependent variable and the independent variables are forced to be monotonic. This is desirable when the overall relationship is not known, because trends can be studied to assist in the development of more precise models. An unfortunate consequence of assuming monotonic relationships is their inability to predict periodic phenomena. To minimize problems associated with choosing correct forms, stepwise regressions can be performed where the model is reduced to lower forms eliminating the effects of the more generalized assumptions. This is done by sorting the data into groups where one variable is held constant and performing the posynomial regression with that variable removed. A fortran algorithm was written to perform a complete stepwise regression for a given generalized relationship (see Appendix B). Three posynomials were regressed;

the first for constant bumper thickness,

$$P^{*} = e^{c_1}\, v^{c_2}\, (\cos\theta)^{c_3}\, d^{c_4} \tag{8}$$

the second for constant obliquity,

$$P^{*} = e^{c_1}\, v^{c_2}\, t_1^{\,c_3}\, d^{c_4} \tag{9}$$

the third for constant bumper thickness and obliquity,

$$P^{*} = e^{c_1}\, v^{c_2}\, d^{c_3} \tag{10}$$

The most complex form of this equation, (6), will provide a very general expression for the ballistic limit; however, this generality is usually gained at the expense of fidelity and, consequently, may fail to produce accurate damage predictions; therefore, all forms should be investigated.

4.3 STATISTICAL ANALYSIS

The statistical routines from which the all statistical parameters were determined, including the ANOVA, were generated from theoretical derivations found in [3] and, subsequently verified by hand calculation and modeling of idealized examples. The Fortran presented in Appendix B includes all statistical formulations presented in this analysis.

The multiple regression program specified correlation coefficient and F statistic only for each stepwise regression fit. This allowed a reasonable determination of the significance of each curve. High correlation coefficients do not always indicate the best fits, they only indicate how well the prediction estimates the observation at the specified position. For higher order polynomials this result is pronounced. Likewise, high values of F statistic may not necessarily indicate a reasonable confidence level. The combination of the two parameters, however, does seem to provide a set of statistical parameters that indicate adequate fits.

5.0 RESULTS AND DISCUSSION

In the following sections, the results of this analysis are presented for both methods of regression. In addition, a comparison is made to the baseline ballistic limits (generated by Boeing) shown in Figure 17, and to the equations proposed by Burch [1] without MLI effects.

5.1 GROUPED LINEAR REGRESSION

Figure 5 shows the results of this method of regression for normal impact of both 0.063" and 0.040" bumpers. The curve associated with a 0.063" bumper compares favorably with the baseline ballistic limit curves and encompasses a sufficient range of velocity. The 0.040" bumper curve, however, differs in shape from the 0.063" curve and does not cover a range of velocity large enough for use in the PNCF analysis (Bumper II).

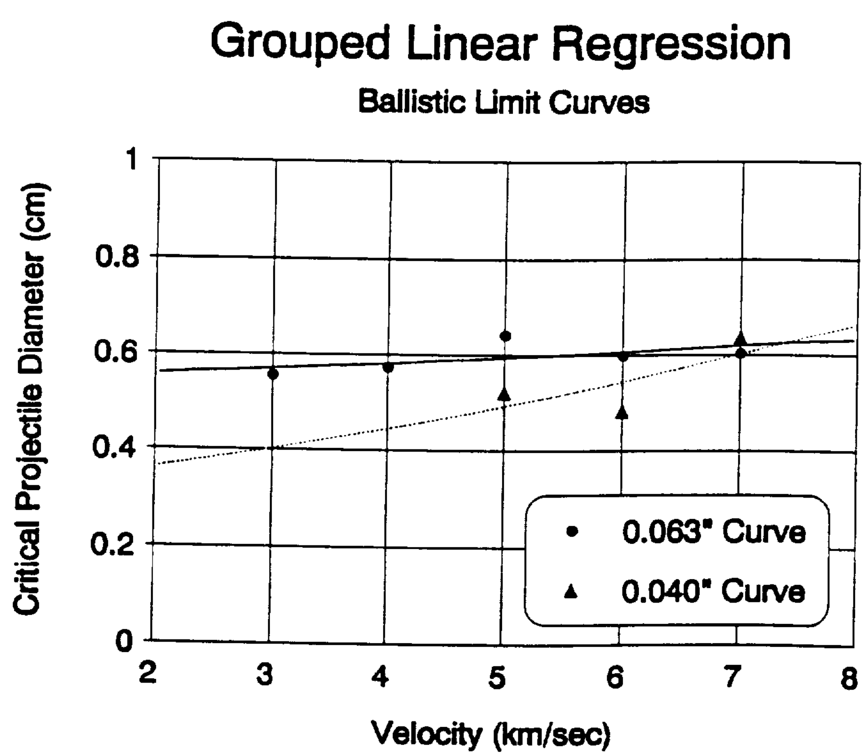

Figure 5. Grouped Linear Regression for Normal Impact on a Dual Wall Target with 0.063" and 0.040" Bumpers

Figures 6-13 are plots of the grouped data for each velocity and for both bumper thicknesses. As is evident in Figures 11-13, the data available for 0.040" bumpers is insufficient to provide statistically significant results. For this reason, it became clear that the amount of data available was not sufficient to generate a complete set of ballistic limit curves using this method; therefore, the analysis was discontinued. It should be noted, however, that when sufficient data is available (e.g. the 0.063" regression), this method does provide a reasonable estimate of the ballistic limit.

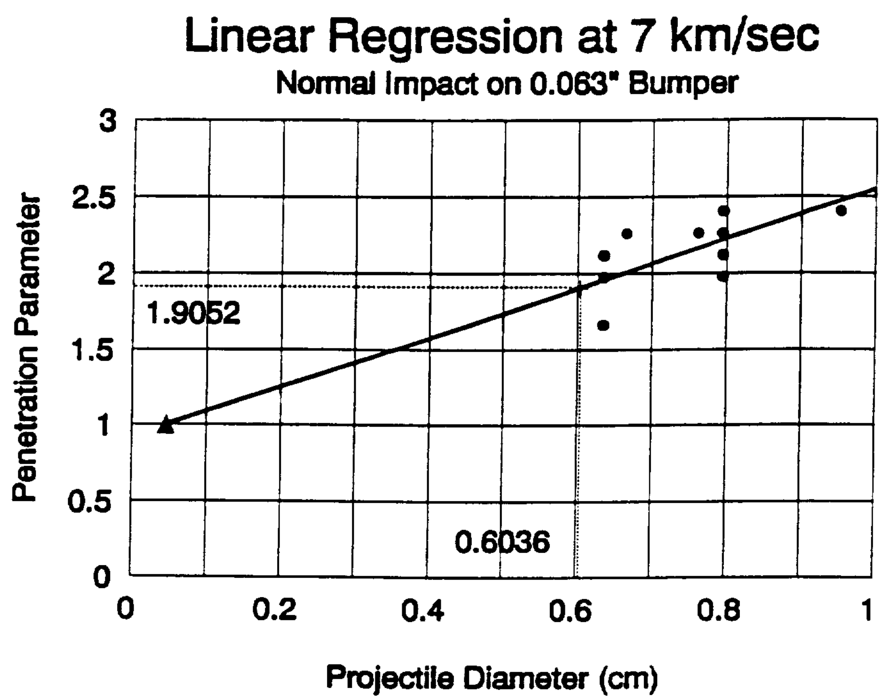

Figure 6. Shots Grouped at 7.0 km/sec for Normal Impact of a Dual Wall Target with a 0.063" Bumper

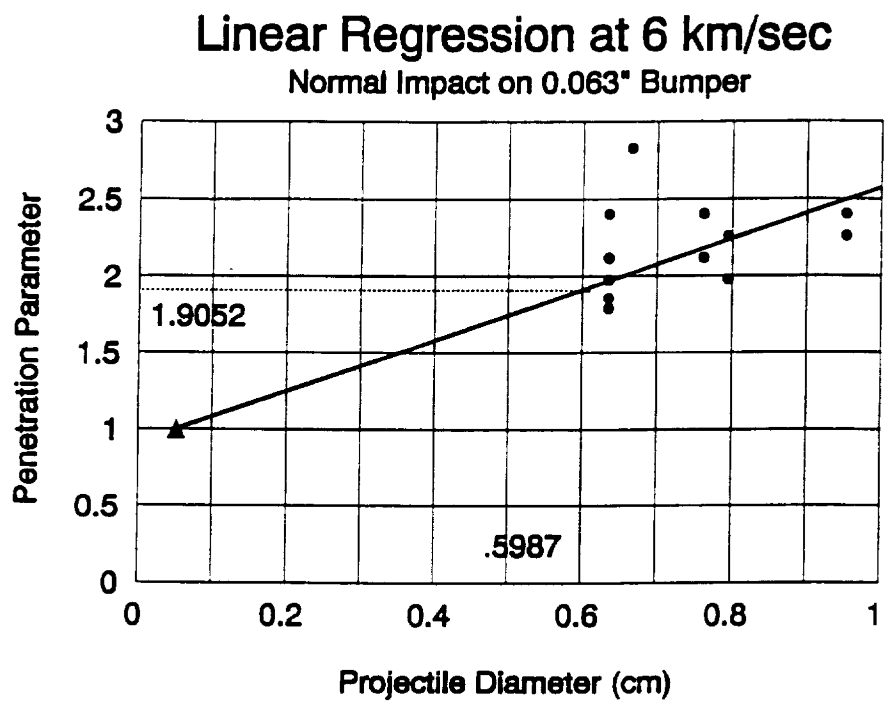

Figure 7. Shots Grouped at 6.0 km/sec for Normal Impact of a Dual Wall Target with a 0.063" Bumper

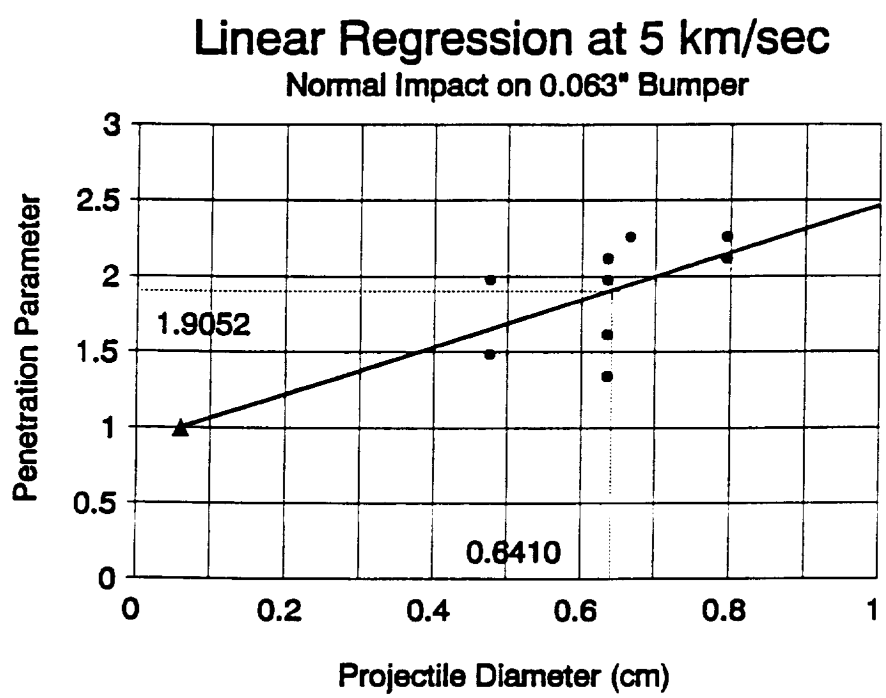

Figure 8. Shots Grouped at 5.0 km/sec for Normal Impact of a Dual Wall Target with a 0.063" Bumper

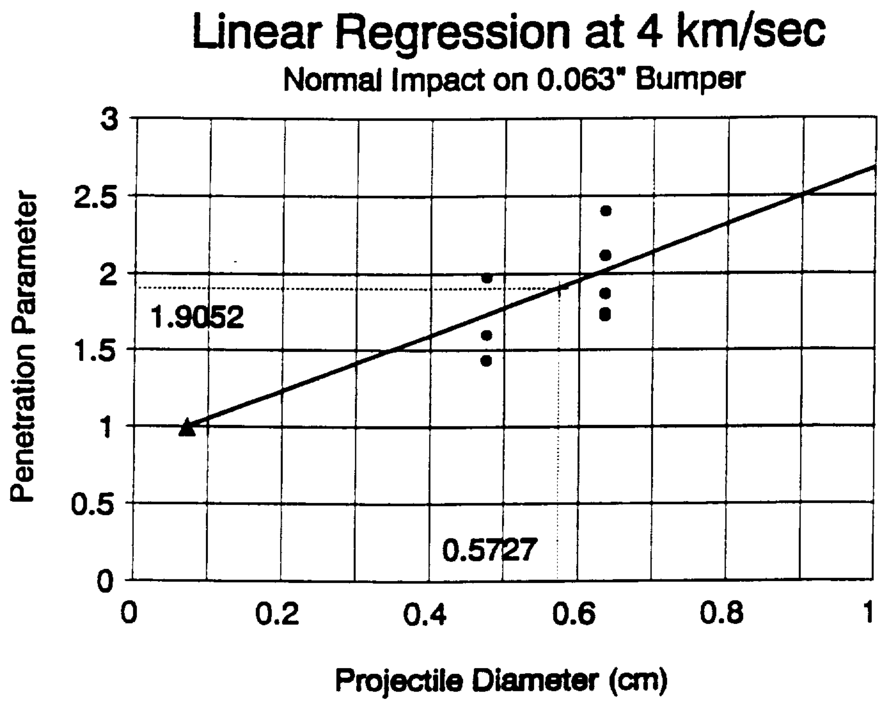

Figure 9. Shots Grouped at 4.0 km/sec for Normal Impact of a Dual Wall Target with a 0.063" Bumper

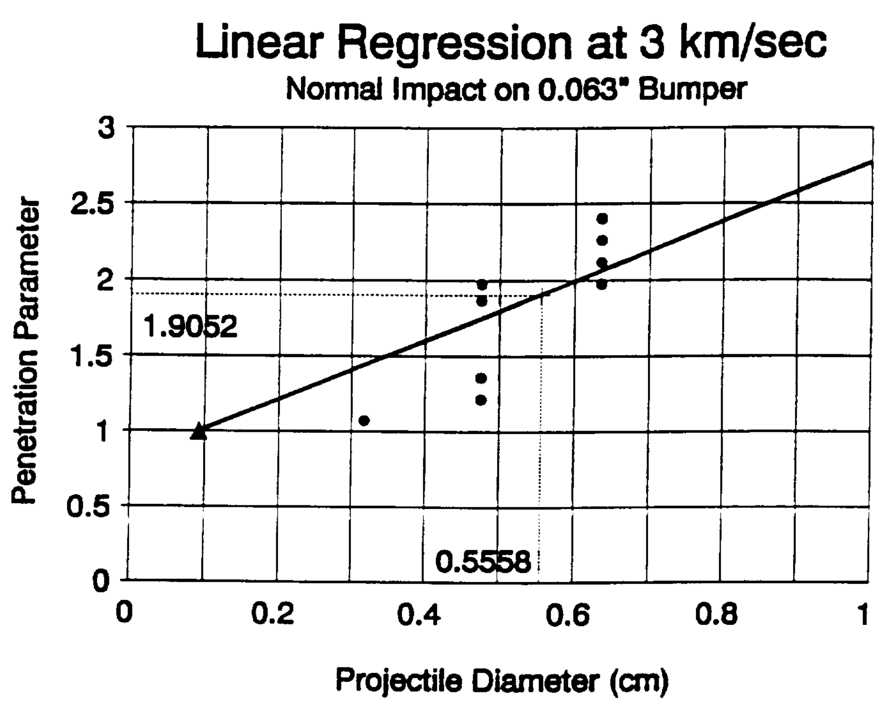

Figure 10. Shots Grouped at 3.0 km/sec for Normal Impact of a Dual Wall Target with a 0.063" Bumper

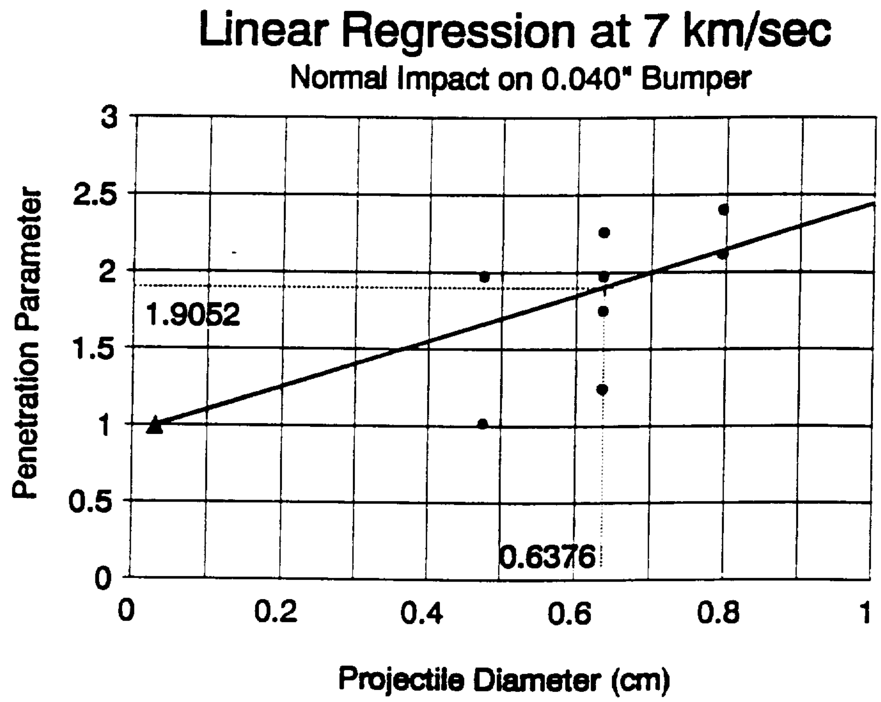

Figure 11. Shots Grouped at 7.0 km/sec for Normal Impact of a Dual Wall Target with a 0.040" Bumper

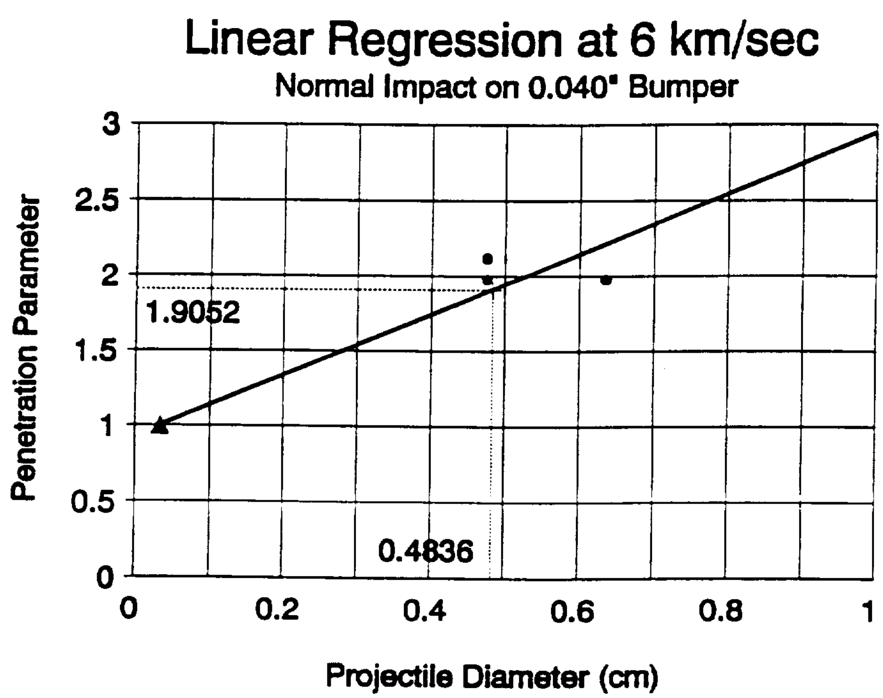

Figure 12. Shots Grouped at 5.75 km/sec for Normal Impact of a Dual Wall Target with a 0.040" Bumper

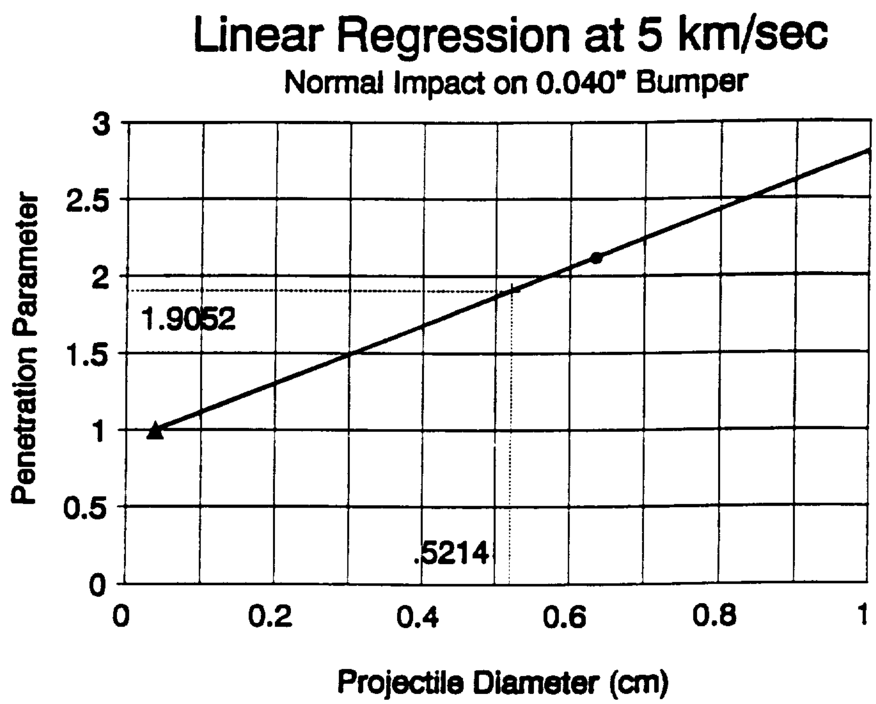

Figure 13. Shots Grouped at 5.0 km/sec for Normal Impact of a Dual Wall Target with a 0.040" Bumper

5.2 MULTIPLE REGRESSION

A generic stepwise regression was performed on the complete set of 385 shots. The results of this regression are presented in Appendix D; however, the shapes of the curves were inconsistent and did not agree with currently accepted theory (i.e., the slope of the velocity curves varied randomly with obliquity and bumper thickness). Inconsistent shapes would not be expected with varying bumper thickness and the velocity exponent for the ballistic limit curve is expected to be positive. Therefore, a detailed study of the data was made by performing a series of regressions on various groupings of the shots.

The model used in the regression was not constructed to include dependence upon the position of the MLI between the shield and the rear wall. Therefore, several regressions were made to study the effects of MLI position in the stack-up. After regressing the sorted data and plotting penetration parameter versus velocity for constant bumper thickness, obliquity, and projectile diameter for various MLI positions, a dependency was established. Tests made with targets having 0.063" bumpers impacted normally with 0.250" projectiles comprised the largest single group of shots. Figure 14 shows this group together with the predicted solution using the applicable equations[4] in [1] and a regression through the associated groups of data. The comparison between the curves indicates the proper functional relationship (or curve shape) results from the regression. Figure 15 is a plot of the regressions of shots with MLI near the bumper, near the rear wall and centered between the walls. This plot indicates that ballistic performance is a function of MLI position and that the presence of MLI tends to reduce the amount of damage incurred by the rear wall. The damage decreases as the distance between the bumper and the rear wall increases.[5] The curves shown in Figure 15 indicate a monotonic relationship between performance and MLI position; therefore, since only a small amount of data exists for targets with MLI centered between the walls, the MLI position parameter could be removed from the model and the entire set of shots where MLI was present could be used to form a regression for the centered configuration (i.e., the average of all the shots should be close to the center curve).

The result of this investigation on the effects of MLI lead to the conclusion that the shots made on targets where MLI was present could be grouped together and a regression made without a parameter for MLI position. Removal of the MLI position parameter from the model was necessary in this analysis because the number of shots with MLI centered is not sufficient to provide significant results.

[4] The Burch equation is plotted to indicate the functional relationship. Since there is no direct means of including MLI in this prediction, the results correspond to the case where MLI is not present in the target configuration.

[5] Although this is true, shots made against targets with MLI placed against the rear wall generally result in massive pedalling failures. These failures are worse than similar events where MLI was not present. Therefore, the current SSF configuration is near optimum with respect to MLI position.

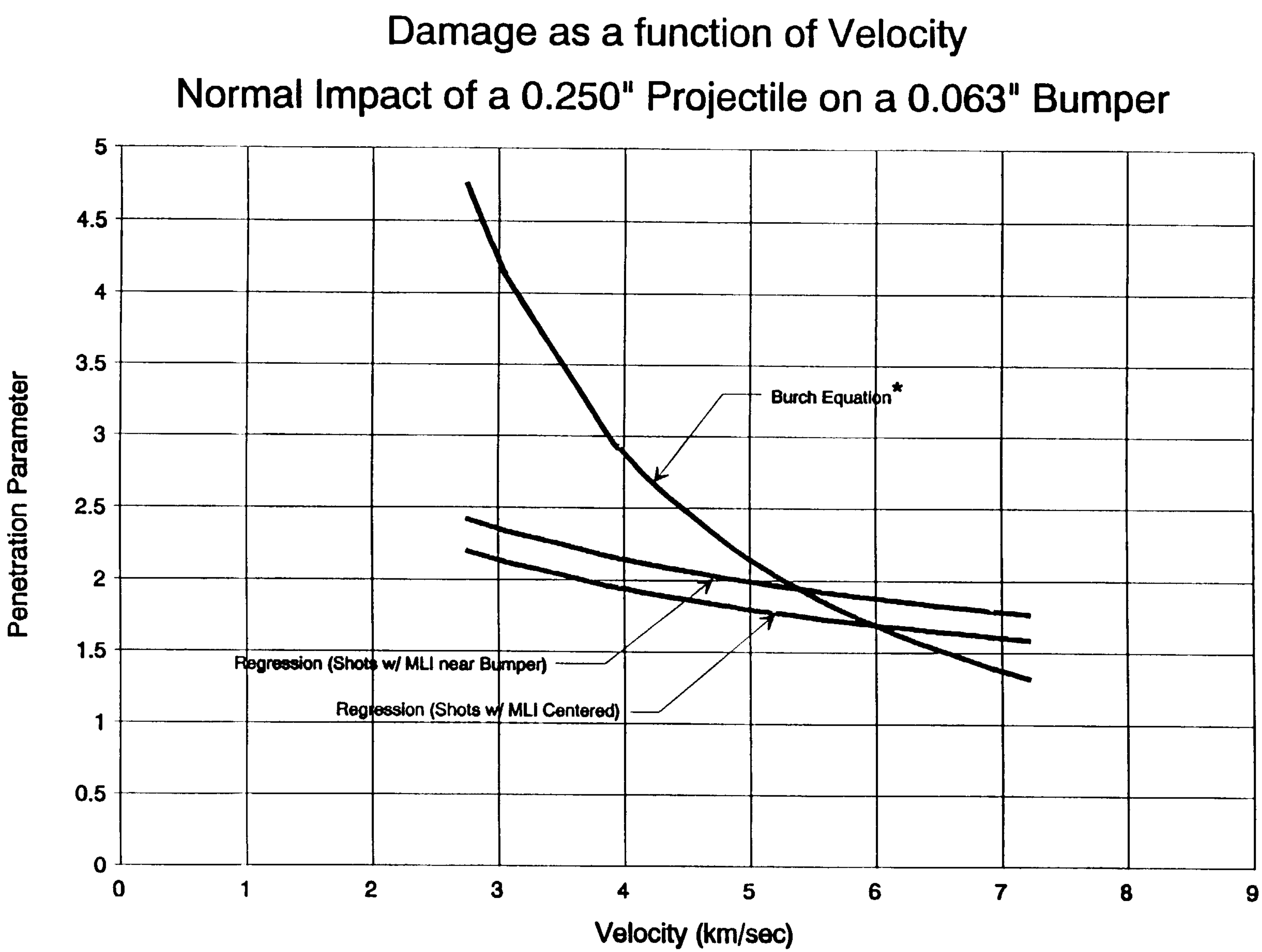

Figure 14. Penetration Parameter versus Velocity for 0.063" Bumper, Normal Impact, and 0.250" Projectile Diameter

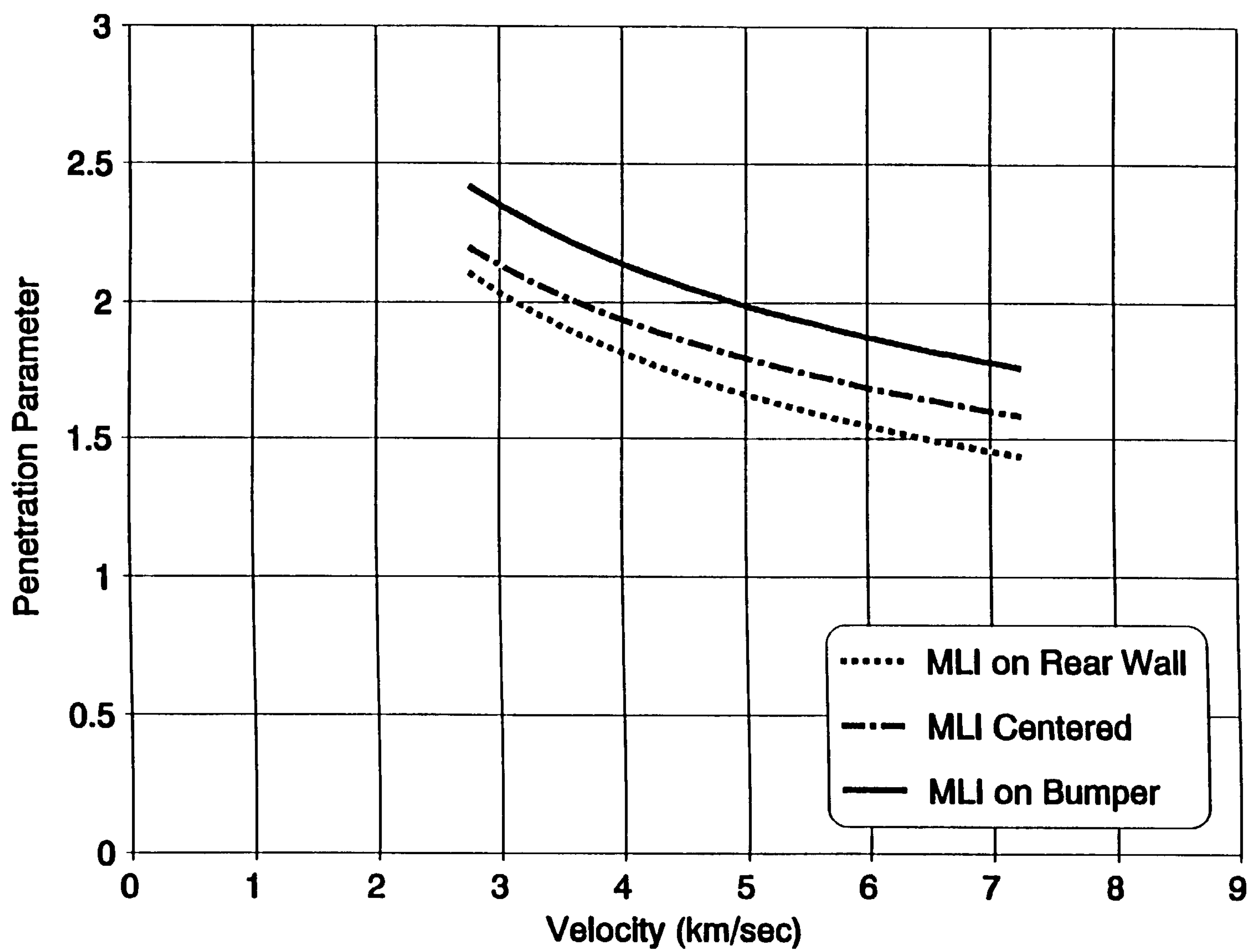

Figure 15. Penetration Parameter versus Velocity for 0.063" Bumper, Normal Impact, 0.250" Projectile Diameter for Three MLI Positions

Another problem noted in the generic regression was the generation of inconsistent results for equations generated from shots made at high obliquities. In studying the high obliquity shot data, the ricochet test series was found to be relatively independent of impact velocity. This appears to be due to the fact that the majority of the shots were fired well in excess of the ballistic limit. This data would, therefore, exhibit a skewed distribution about a ballistic limit function and violate the normal distribution assumption necessary for the derivation of the least squares regression.

These anomalies were remedied by filtering the data. Keeping shots fired at 0°, 45°, and 65° obliquities and discarding shots where MLI was not present, reduced the total number of shots used in the regression to 221. The new coefficients generated from another stepwise regression of the remaining data are presented in Appendix C.1.

Figure 15 illustrates one set of BLCs suggested by the analysis corresponding to the more general equation:

$$P^* = e^{0.8533} \, v^{-0.0547} \, t_1^{-0.0815} \, (\cos\theta)^{0.2238} \, d^{0.5268} \tag{11}$$

Substituting $P^* = P_c^* = 1.9052$ and solving for the projectile diameter results in the ballistic surface described by:

$$d_c = 0.6729 \, v^{0.1038} \, t_1^{0.1546} \, (\cos\theta)^{-0.4249} \tag{12}$$

Figure 16 was generated using equation **(12)** with the bumper thickness set to 0.050" to represent the Space Station dual wall configuration.

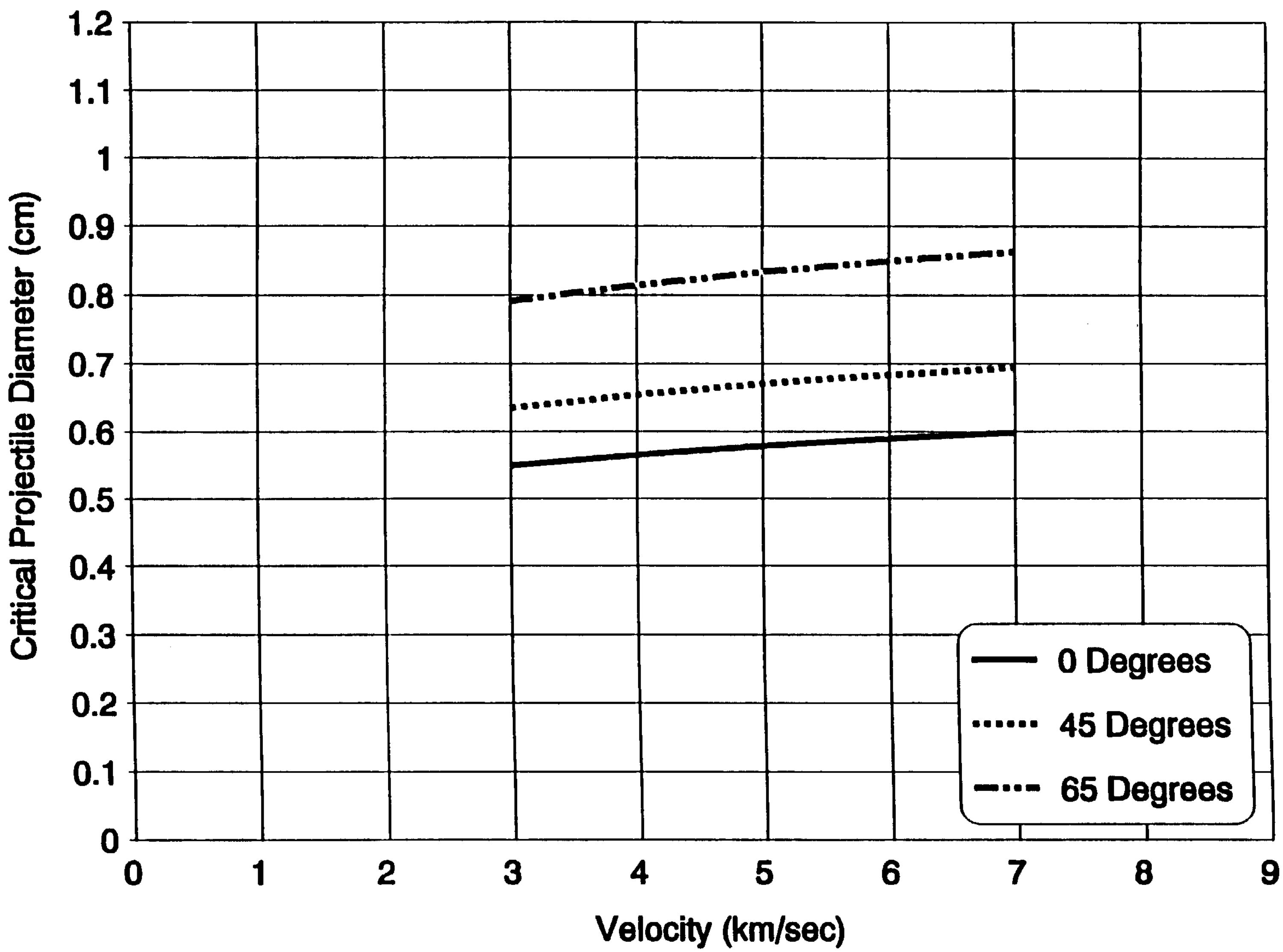

Figure 16. Generalized Regression Curves

As stated previously, the generality of this equation may reduce its accuracy, particularly with respect to obliquity. Therefore, another set of BLCs were regressed, one for each obliquity; for normal impacts,

$$P^* = e^{0.6160}\, v^{-0.1699}\, t_1^{-0.2977}\, d^{0.5694} \tag{13}$$

for 45° impacts,

$$P^* = e^{0.7627}\, v^{-0.0333}\, t_1^{-0.1605}\, d^{0.7783} \tag{14}$$

for 65° impacts,

$$P^* = e^{1.3686}\, v^{-0.1137}\, t_1^{0.2218}\, d^{0.5726} \tag{15}$$

Substituting $P^* = P_c^* = 1.9052$ and solving for the projectile diameter results in a set of ballistic limit curves defined by the following equations;

$$d_c = 1.0514\, v^{0.2983}\, t_1^{0.5228} \tag{16}$$

$$d_c = 0.8591\, v^{0.0428}\, t_1^{0.2063} \tag{17}$$

$$d_c = 0.2824\, v^{0.1986}\, t_1^{-0.3874} \tag{18}$$

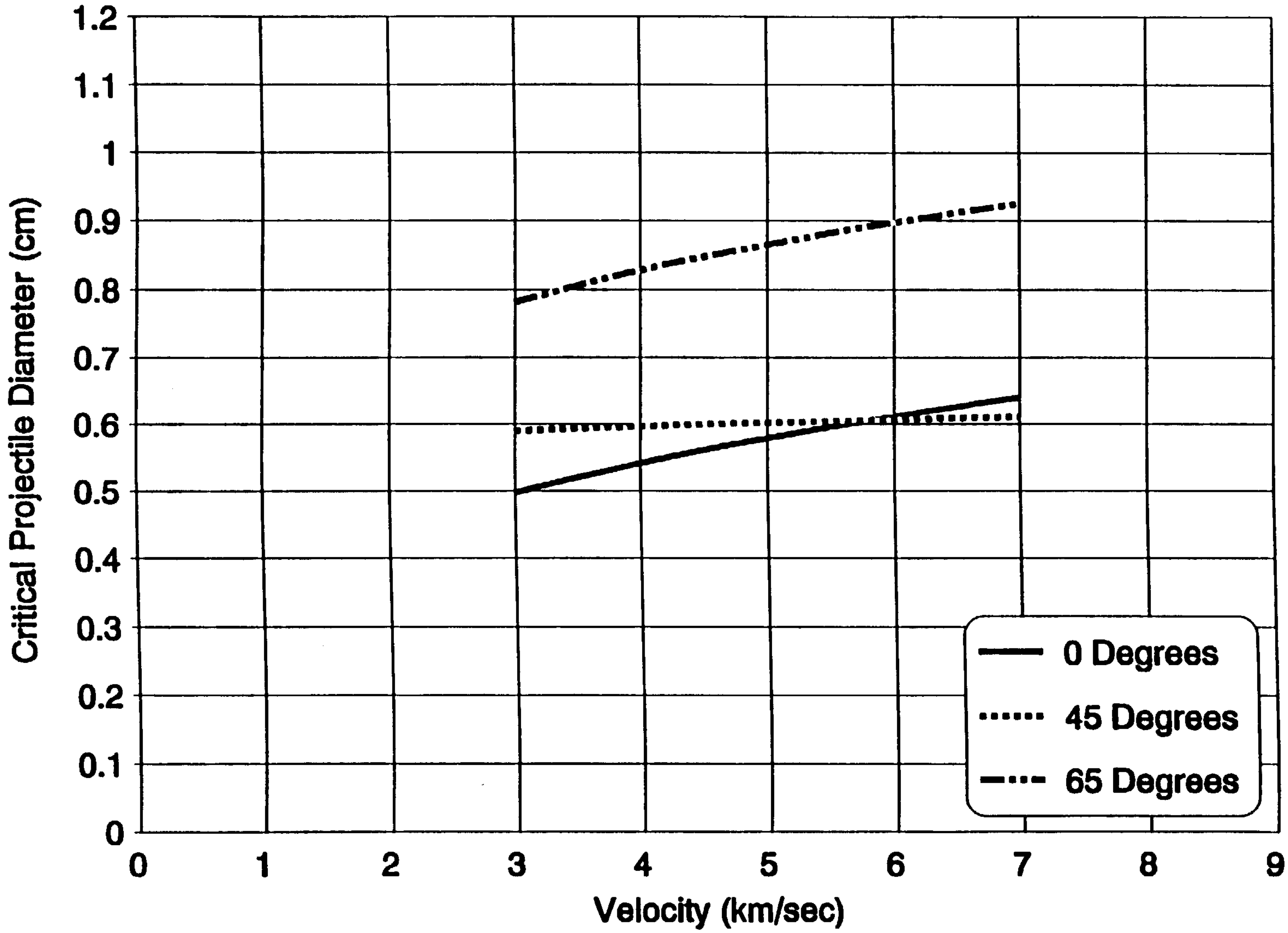

Figure 17. Ballistic Limit Curves Regressed at Constant Obliquity

These functions are illustrated in Figure 17. Equations (11) through (18) are valid for the dual wall protection system shown in Figure 1 with bumpers between 0.032" and 0.080" thick impacted by aluminum spheres at velocities between three and seven km/sec.

In an effort to choose the most accurate expression for use in the determination of PNCF, a brief study of the dataset was made to look for shots that might indicate a ballistic limit. A series of three tests was found where 0.187" projectiles impacted a 0.063" bumper at 0° obliquity with velocities between 3.9 and 4.5 km/sec. This velocity range appears to be very near the ballistic limit because in two cases the rear wall was penetrated without penetrating witness plates and in the third case 50% of the rear wall was penetrated. If we assume a ballistic limit for a 0.475 cm projectile to be ~4.0 km/sec, then the more general expression makes a better prediction of 0.5883 cm, as compared to the normal impact equation's prediction of 0.6189 cm. Both regressions are noted as being anti-conservative. It must be understood that these are P_{50} (or 50% probability of prediction) curves and that the lower bounds provide estimations based on the confidence intervals.

5.2.1 PNCF Sensitivity Study

The program code Bumper II was used to determine the effects of using the new constant obliquity curves in the PNCF analysis. The full analysis of Space Station was not performed, instead a series of runs using the MB-7 build configuration was performed incrementally over 25 years. A negligible increase in probability of no penetration (PNP) of 0.17% was the effect of using the new equations after an exposure time of 25 years. The specific results of this series of computer runs are presented in Appendix F.

5.3 STATISTICAL SIGNIFICANCE

All regressions made had statistical parameters generated for them; however, the full ANOVA was reserved for only the final set of equations, (11) through (18). The correlation coefficients and

F statistics for all of the regressions are provided in Appendix C along with the model coefficients. The F value of 23.064 for the generalized regression is in excess of 5.63, the 5% level of significance value for the F-distribution, which allows the rejection of the null hypothesis. The corresponding value of the correlation coefficient for 50 degrees of freedom and 4 predictor variables is .379 for 5% level of significance and .449 for 1% level of significance. This relates to $r = \sqrt{0.299} = .547$ for the generalized regression ($r^2 = .299$) which indicates adequate fit for the number of variables involved. Table 7 is a compilation of similar values for the constant obliquity regressions.

Table 7. Comparison Statistics Parameters

Regression Equation	F-Distribution Value	Correlation Coefficient (r)	
	Upper 5%	5% Significance	1% Significance
Generalized	5.63	.379	.449
Constant Obliquity 0°	8.56	.336	.410
Constant Obliquity 45°	8.56	.336	.410
Constant Obliquity 65°	8.61	.397	.481

Tables 8 through 11 provide statistical parameters for each regression equation presented. Residual plots for each model are presented in Appendix D.

Table 8. ANOVA for Generalized Regression

Source	Degrees of Freedom	SS	MS	F Value
Regression	4	2.821	0.705	23.064
Residual	216	6.605	0.031	
Total Corrected	220	9.426		
Multiple Correlation Coefficient (r^2) =		0.299	(r = .547)	
Reduced Ballistic Equation Multiplier =		1.045	(95% Confidence Interval)	

Table 9. ANOVA for 0° Constant Obliquity Regression

Source	Degrees of Freedom	SS	MS	F Value
Regression	3	0.655	0.218	6.001
Residual	85	3.095	0.036	
Total Corrected	88	3.751		
Multiple Correlation Coefficient (r^2) =		0.175	(r = .418)	
Reduced Ballistic Equation Multiplier =		1.073	(95% Confidence Interval)	

Table 10. ANOVA for 45° Constant Obliquity Regression

Source	Degrees of Freedom	SS	MS	F Value
Regression	3	2.348	0.782	55.402
Residual	92	1.300	0.014	
Total Corrected	95	3.647		
Multiple Correlation Coefficient (r^2) =		0.644	(r = .802)	
Reduced Ballistic Equation Multiplier =		1.031	(95% Confidence Interval)	

Table 11. ANOVA for 65° Constant Obliquity Regression

Source	Degrees of Freedom	SS	MS	F Value
Regression	3	0.479	0.160	6.428
Residual	32	0.795	0.025	
Total Corrected	35	1.274		
Multiple Correlation Coefficient (r^2) =		0.376	(r = .613)	
Reduced Ballistic Equation Multiplier =		1.098	(95% Confidence Interval)	

5.4 BASELINE BALLISTIC LIMITS

Figure 18 is an interpolation of the ballistic limit curves currently used to calculate PNCF for SSF. These curves are proposed for use in [9] and are presented here to indicate the relative shift in the ballistic limit proposed by this analysis for Space Station protective structures.

An alternative viewpoint is that this analysis may be viewed as a verification of the baseline curves.

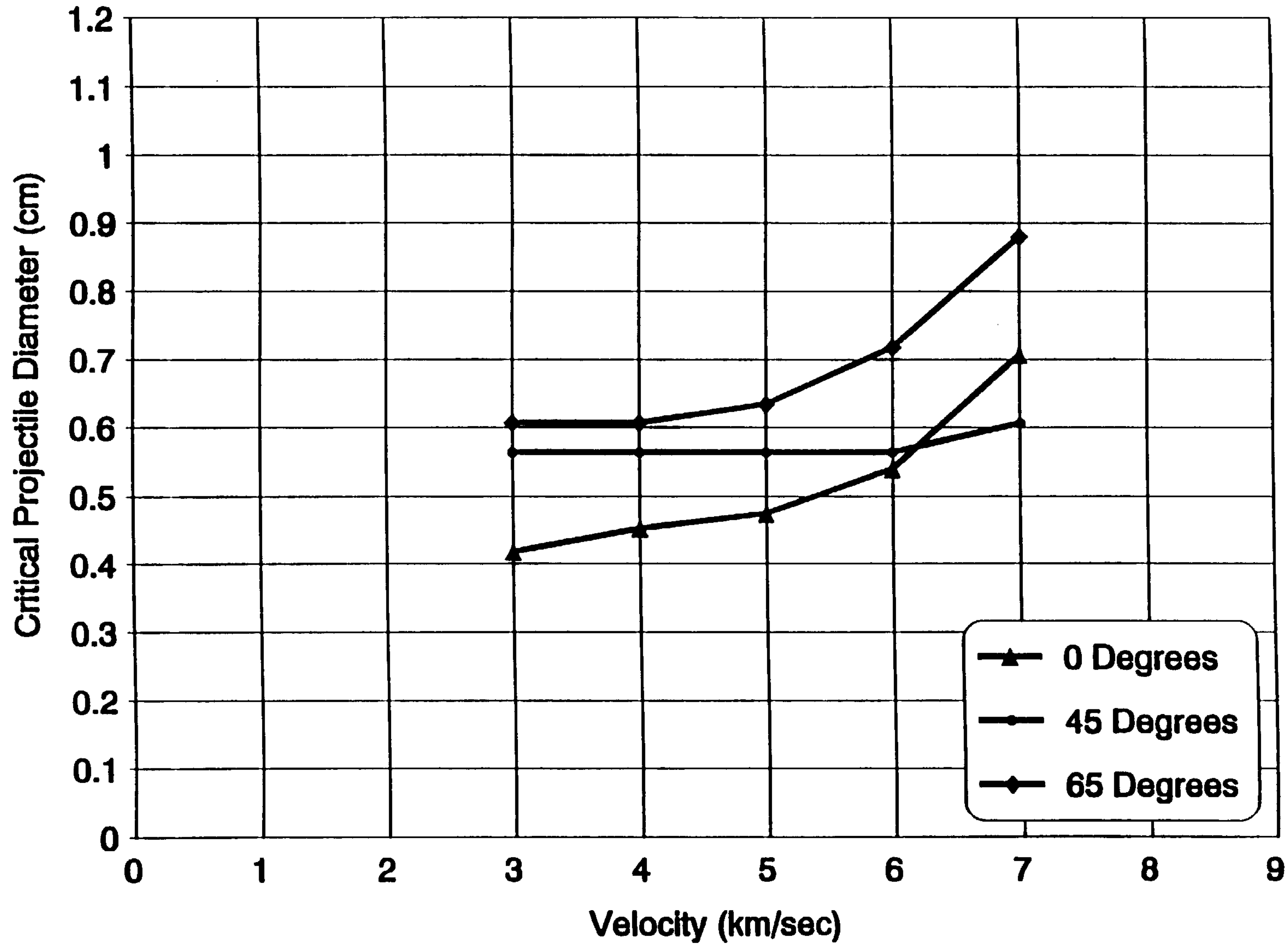

Figure 18. Baseline Ballistic Limit Curves Interpolated for a 0.050" Bumper

6.0 CONCLUSIONS AND RECOMMENDATIONS

The following sections contain some of the conclusions that can be drawn from the data studied during this analysis. The recommendations presented here are given to assist in the selection of future shots made at the Light Gas Gun Facility, MSFC that would enhance the accuracy and statistical significance of curves generated by similar methods in future regression analyses.

6.1 DUAL WALL BALLISTIC LIMIT CURVE

The curves shown in Figure 17, where obliquity was held constant in the regression model appear to match the curves generated by Boeing (Figure 18). These curves are recommended for use as limit curves for Space Station Freedom protection systems. They may, on the other hand, be considered as verification of baseline curves because of similarity in the predicted diameters.

The generalized curves, shown in Figure 16, indicate closer agreement with Burch's expressions with respect to the sign and magnitude of the velocity exponent and indicate better overall statistical variance. The major difference between the sets of equations is how the target performance varies with obliquity. In the generalized curves, performance increases monotonically with obliquity. The curves regressed over constant obliquity indicate that a monotonic relationship may not be correct and are therefore preferred over the generalized regressions.

Another observation is that the constant obliquity curves are more conservative than the generalized curves at lower obliquities but both are anti-conservative when compared to ballistic limits indicated by the results of specific shots.

6.2 STATISTICAL SIGNIFICANCE

Confidence intervals have been defined for the final set of ballistic limit curves and are presented along with the regression results for each expression in Appendix C.2. The constant obliquity predictor curves have 95% confidence intervals all within 10%[6] of the mean curve at the mean location (mean vectors are also presented in Appendix C.1). Considering the random nature of this event, an interval within 10% is acceptable for the regression; however, it must be noted that the percentage represents the interval at the mean and is therefore a minimum.

6.3 CONTINUATION OF ANALYSIS

Many other models are possible candidates for comparison against the data presented in this analysis; however they do not lend themselves to least squares regression techniques. Therefore, full non-linear analyses may provide useful information leading to more general expressions. In addition, greater insight into the phenomenology of the effects of obliquity on penetration of dual wall systems would provide higher confidence in those more generalized models.

6.4 ADDITIONAL TESTING

The following shots are recommended for future hypervelocity impact tests to provide additional data for use in improving accuracy of BLCs generated by regressive techniques. All future SSF shots should be made against targets with MLI centered between the walls and on 0.050" bumpers to reflect the actual SSF protection system configuration. In addition, more shots between 0 and 45 degrees are needed to characterize the system performance with respect to obliquity.

[6] The percentage given is a rounded value of the reduced regression multiplier. Statistically, this indicates that there is a 95% confidence that a mean predicted critical diameter of .5 cm would fall within .45 and .55 cm.

Table 12. Shot Parameters for Recommended Testing.

Impact Velocity (km/sec)	Projectile Diameter (cm)	Obliquity (degrees)
5	0.313	0
		30
		45
	0.250	0
		30
		45
6	.0313	0
		30
		45
	0.250	0
		30
		45
7	0.313	0
		30
		45
	0.250	0
		30
		45

Total Number of Recommended Shots = 18

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5. N. R. Draper and H. Smith, "Applied Regression Analysis" second ed., John Wiley and Sons, 1981, pp 141.

6. Space Station Program Natural Environment Definition for Design, JSC 30425.

7. Robert A. Mog, "Posynomial Regression Analysis for Hypervelocity Impact Prediction", White Paper prepared for George C. Marshall Space Flight Center, September 1989, Science Applications International Corporation.

8. Space Station Freedom Program Definition and Requirements, SS-SRD-0001C, SEC. 3.0, 17 April 1990, Marshall Space Flight Center, Huntsville, Alabama.

9. Boeing Areospace Company, "Results of the Ballistic Limit Testing of Aluminum M/D Shields," Document No. D683-10578-1, Preliminary.

APPENDICES

APPENDIX A

REGRESSION DATA

APPENDIX A.1 - 385 SHOTS - SPACE STATION CONFIGURATION

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize

bumallo	rwalloy	rwgage	prjmtrl	stdoff	witpen
6061-T6	2219-T8	0.125	1100-O	4	<>99.0

385 Datapoints - Space Station Configuration

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
43	SS-P-001	0	0.125	N	Y	2.75	0.063	0	0.25
44	SS-P-002	1	0.125	N	Y	2.99	0.063	0	0.25
46	SS-P-004	1	0.125	N	Y	4.95	0.063	0	0.25
47	SS-P-005	0	0.125	N	Y	6.9	0.063	0	0.25
49	SS-P-006A	0	0.125	N	Y	6.95	0.063	0	0.25
50	SS-P-007	0	0.125	W	Y	2.93	0.063	0	0.25
51	SS-P-008	1	0.125	W	Y	2.96	0.063	0	0.25
115	SS-P-027	0	0.067	N	N	4.53	0.063	0	0.187
116	SS-P-027A	0	0.125	N	Y	3.87	0.063	0	0.187
117	SS-P-027B	0	0.125	N	Y	4.15	0.063	0	0.187
118	SS-P-027C	0	0.06	W	N	3.68	0.063	0	0.187
119	SS-P-027D	0	0.03	W	N	3.08	0.063	0	0.187
120	SS-P-027E	0	0.05	W	N	2.83	0.063	0	0.187
121	SS-P-028	0	0.01	N	N	3	0.063	0	0.125
123	SS-P-012C	0	0.1	W	N	4.33	0.063	0	0.25
124	SS-P-012D	0	0.12	W	N	3.96	0.063	0	0.25
125	SS-P-027F	0	0.12	W	N	2.54	0.063	0	0.187
126	SS-P-021	2	0.125	N	Y	6.63	0.063	0	0.3
127	SS-P-021A	3	0.125	N	Y	6.47	0.063	0	0.3
128	SS-P-021B	0	0.01	W	N	6.89	0.063	0	0.3
129	SS-P-021C	0	0.01	W	N	6.6	0.063	0	0.3
130	SS-P-021D	1	0.125	W	Y	5.85	0.063	0	0.3
131	SS-P-022	2	0.125	N	Y	5.09	0.063	0	0.262
132	SS-P-022A	6	0.125	W	Y	6.16	0.063	0	0.262
133	SS-P-022B	2	0.125	N	Y	6.89	0.063	0	0.262
134	SS-101	2	0.125	N	Y	3.094	0.08	0	0.187
135	SS-101A	1	0.125	N	Y	3.696	0.08	0	0.187
136	SS-101B	0	0.125	N	C	4.27	0.08	0	0.187
137	SS-102	0	0.125	N	Y	7.2	0.08	0	0.3
138	SS-102A	0	0.02	W	N	5.35	0.08	0	0.3
139	SS-102B	0	0.01	W	N	5.96	0.08	0	0.3
140	SS-102C	0	0.125	W	C	4.74	0.08	0	0.3
141	SS-102D	0	0.125	W	Y	3.83	0.08	0	0.3
149	SS-105	0	0.125	N	Y	3.51	0.08	45	0.35
150	SS-105A	1	0.125	N	Y	4.05	0.08	60	0.35
151	SS-105B	0	0.001	N	N	3.89	0.08	75	0.35
153	SS-109	0	0.05	N	N	7.39	0.08	0	0.187
155	SS-107	4	0.125	N	Y	6.8	0.08	0	0.35
157	SS-106	6	0.125	N	Y	6.84	0.08	45	0.35
158	SS-106-1	1	0.125	N	Y	6.8	0.08	60	0.35
159	SS-106-2	0	0.125	N	Y	6.65	0.08	75	0.35
160	SS-106A	3	0.125	N	Y	6.66	0.08	60	0.375
161	SS-106B	2	0.125	N	Y	6.73	0.08	75	0.375

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
164	SS-109B	0	0.088	N	N	3.61	0.08	0	0.187
166	SS-109D	0	0.08	N	N	2	0.08	0	0.187
175	SS-114B	3	0.125	N	Y	3.51	0.032	45	0.3
176	SS-P-034B	0	0.125	N	Y	7.06	0.063	0	0.25
181	SS-P-033	2	0.125	N	Y	7.21	0.04	0	0.25
182	SS-P-033B	1	0.125	W	Y	4.85	0.04	0	0.25
183	SS-P-033B	1	0.125	N	Y	5.26	0.04	0	0.25
184	SS-P-033C	0	0.125	N	C	5.53	0.04	0	0.25
214	SS-202A	0	0.125	N	Y	3.52	0.04	45	0.187
215	SS-202B	1	0.125	N	Y	3.24	0.04	45	0.187
217	SS-206A	1	0.125	N	Y	4.77	0.063	45	0.187
218	SS-206B	0	0.135	N	N	5.09	0.063	45	0.187
219	SS-206C	0	0.125	N	Y	5.4	0.063	45	0.187
220	SS-206D	0	0.125	N	C	3.69	0.063	45	0.187
226	SS-208A	0	0.087	N	N	4.98	0.063	65	0.25
227	SS-208B	0	0.125	N	Y	4.29	0.063	65	0.25
228	SS-208C	0	0.084	N	N	3.32	0.063	65	0.25
229	SS-208D	0	0.125	N	C	5.63	0.063	65	0.25
230	SS-206E	0	0.084	N	N	3.24	0.063	45	0.187
231	SS-206F	0	0.069	N	N	6.15	0.063	45	0.187
238	SS-205A	0	0.125	W	Y	4.16	0.063	45	0.25
239	SS-205B	0	0.125	W	Y	4.61	0.063	45	0.25
240	SS-205C	1	0.125	W	Y	5.3	0.063	45	0.25
241	SS-205D	0	0.125	W	C	6.3	0.063	45	0.25
242	SS-205E	2	0.125	W	Y	3.15	0.063	45	0.25
245	SS-204A	0	0.125	N	Y	4.77	0.04	65	0.25
246	SS-204B	0	0.125	N	C	5.86	0.04	65	0.25
247	SS-204C	0	0.075	N	N	4.25	0.04	65	0.25
249	SS-207A	0	0.125	W	Y	5.74	0.063	65	0.3
253	SS-203A	0	0.056	W	N	4.79	0.04	65	0.3
254	SS-203B	0	0.126	W	N	3.65	0.04	65	0.3
255	SS-203C	0	0.065	W	N	2.72	0.04	65	0.3
256	SS-203D	0	0.041	W	N	5.59	0.04	65	0.3
259	SS-202C	2	0.125	N	Y	5.25	0.04	45	0.187
261	SS-202D	1	0.125	N	Y	6.44	0.04	45	0.187
262	SS-202E	0	0.125	N	Y	7.19	0.04	45	0.187
263	SS-202F	0	0.125	N	Y	7.51	0.04	45	0.187
264	SS-208E	1	0.125	N	Y	6.47	0.063	65	0.25
265	SS-204D	0	0.065	N	N	3.18	0.04	65	0.25
266	SS-201A	1	0.125	W	Y	4.33	0.04	45	0.25
267	SS-201B	4	0.125	W	Y	5.51	0.04	45	0.25
268	SS-201C	0	0.125	W	Y	7.21	0.04	45	0.25
269	SS-201D	2	0.125	W	Y	7.59	0.04	45	0.25
270	SS-203E	0	0.077	W	N	6.72	0.04	65	0.3
271	SS-207B	0	0.125	W	Y	6.25	0.063	65	0.3
273	SS-207C	0	0.049	W	N	7.03	0.063	65	0.3
274	SS-209A	0	0.072	W	N	4.29	0.063	65	0.25
275	SS-209B	0	0.074	W	N	6.35	0.063	65	0.25
276	SS-209D	0	0.102	W	N	7.34	0.063	65	0.25
277	SS-203F	0	0.125	W	C	3.05	0.04	65	0.35
278	SS-203G	0	0.125	W	Y	4.64	0.04	65	0.35
280	SS-210B	0	0.125	W	Y	5.69	0.063	65	0.35
281	SS-210D	1	0.125	W	Y	6.93	0.063	65	0.35
282	SS-211B	4	0.125	W	Y	5.87	0.063	45	0.35

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
283	SS-211D	4	0.125	W	Y	6.97	0.063	45	0.35
284	SS-212B	2	0.125	W	Y	6.27	0.063	45	0.3
303	SS-135A	2	0.125	N	Y	5.86	0.063	30	0.25
304	SS-135B	0	0.125	N	Y	7.18	0.063	30	0.25
305	SS-135C	1	0.125	N	Y	6.67	0.063	30	0.25
306	SS-135D	2	0.125	N	Y	6.86	0.063	30	0.25
307	SS-135E	0	0.125	N	Y	7.21	0.063	30	0.25
308	SS-136A	1	0.125	N	Y	6.25	0.063	55	0.25
309	SS-136B	0	0.125	N	C	7.24	0.063	55	0.25
310	SS-136C	1	0.125	N	Y	6.67	0.063	55	0.25
336	SS-EH1A	4	0.125	N	Y	7.11	0.063	30	0.313
337	SS-EH1B	4	0.125	N	Y	7.01	0.063	45	0.313
338	SS-EH1C	0	0.125	N	Y	7.17	0.063	60	0.313
340	SS-EH1D	1	0.125	N	Y	7.16	0.063	75	0.313
344	SS-221A	0	0.046	W	N	6.42	0.04	45	0.187
345	SS-221B	0	0.03	W	N	5.93	0.04	45	0.187
346	SS-221C	0	0.084	W	N	4.6	0.04	45	0.187
347	SS-221D	0	0.125	W	c	4.08	0.04	45	0.187
357	SS-222A	0	0.054	N	N	5.6	0.04	45	0.125
358	SS-222B	0	0.048	N	N	5.03	0.04	45	0.125
359	SS-222C	0	0.051	N	N	3.29	0.04	45	0.125
385	SS-230A	0	0.068	W	N	4.41	0.063	45	0.187
386	SS-230B	0	0.04	W	N	3.23	0.063	45	0.187
445	SS-150A	2	0.125	N	Y	7	0.063	45	0.25
448	MD-TEST-B	0	0.125	W	C	2.45	0.063	0	0.187
453	SS-230C	3	0.125	N	Y	5.18	0.063	45	0.25
456	SS-230D	2	0.125	N	Y	5.55	0.063	45	0.25
457	SS-230E	1	0.125	N	Y	6.57	0.063	45	0.25
461	SS-151A	0	0.125	N	Y	6.88	0.08	45	0.25
463	SS-001A	2	0.125	N	Y	6.62	0.08	45	0.313
464	SS-001B	0	0.125	W	Y	6.53	0.08	45	0.313
466	SS-002A	4	0.125	N	Y	6.5	0.063	45	0.313
467	SS-002B	0	0.125	W	Y	6.45	0.063	45	0.313
485	MD-TEST-D	0	0.125	W	Y	5.63	0.063	0	0.313
497	SS-003A	4	0.125	W	Y	6.54	0.04	45	0.313
503	SS-154A	0	0.001	N	N	6.83	0.04	45	0.187
504	SS-154B	0	0.125	N	Y	5.95	0.04	45	0.187
508	SS-231A	0	0.001	N	N	3.34	0.063	65	0.187
509	SS-231B	0	0	N	N	2.44	0.063	65	0.187
510	SS-155A	0	0.125	N	Y	7.02	0.063	45	0.187
512	SS-231D	0	0.125	N	Y	7.26	0.063	65	0.313
517	SS-157A	0	0.001	N	N	7.4	0.063	60	0.187
524	SS-319	4	0.125	W	Y	2.99	0.04	45	0.313
525	SS-320	0	0.125	W	Y	3.08	0.063	45	0.313
526	SS-321	2	0.125	W	Y	3.01	0.08	45	0.313
527	SS-324	4	0.125	W	Y	4.12	0.04	45	0.313
528	SS-325	1	0.125	W	Y	4.25	0.063	45	0.313
529	SS-326	3	0.125	W	Y	4.25	0.08	45	0.313
531	SS-335	1	0.125	W	Y	4.12	0.04	45	0.25
532	SS-336	2	0.125	W	Y	4.54	0.04	45	0.25
533	SS-334	0	0.109	W	N	3.66	0.04	45	0.187
535	SS-333	0	0.098	W	N	2.93	0.04	45	0.187
536	SS-336A	0	0.125	W	C	5.76	0.04	45	0.25
538	SS-338	2	0.125	W	Y	7.02	0.04	45	0.313

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
540	SS-337	4	0.125	W	Y	6.9	0.04	45	0.313
544	SS-339	3	0.125	W	Y	6.49	0.04	45	0.375
545	SS-162A	0	0.125	N	S	6.49	0.063	30	0.187
546	SS-162B	0	0.125	N	Y	5.11	0.063	30	0.187
593	EH1-A	1	0.125	N	Y	6.9	0.063	75	0.313
594	EH1-AA	1	0.125	N	Y	6.94	0.063	75	0.313
599	SS-303A	2	0.125	W	Y	3.72	0.063	45	0.313
636	EH3-A	2	0.125	N	Y	6.64	0.063	0	0.313
675	EH4-A	2	0.125	W	Y	6.13	0.063	0	0.313
676	EH4-B	2	0.125	N	Y	6.76	0.063	0	0.313
680	EHSS-2A	0	0.125	N	S	6.36	0.063	0	0.25
681	EHSS-2B	1	0.125	N	Y	5.88	0.063	0	0.25
691	EHSS-3C	2	0.125	N	Y	6.81	0.063	30	0.25
703	EHSS-6C	2	0.125	N	Y	6.64	0.063	0	0.313
717	EHRP-7	0	0.125	N	Y	8.04	0.063	60	0.187
720	EHRP-8	0	0.125	N	Y	7.39	0.063	45	0.187
798	3001-A	2	0.125	W	Y	3.99	0.032	0	0.187
799	3001-B	1	0.125	W	Y	5.78	0.032	0	0.187
800	3001-C	0	0.125	W	Y	6.27	0.032	0	0.187
802	3001-E	0	0.125	W	Y	6.82	0.032	0	0.187
804	3002-B	2	0.125	W	Y	7.39	0.032	0	0.25
808	3005-A	0	0.125	W	Y	7.46	0.032	45	0.25
810	3004-B	0	0.125	W	Y	6.64	0.032	45	0.187
811	3005-B	0	0.094	W	N	7.69	0.032	45	0.25
812	3006-A	3	0.125	W	Y	7.12	0.032	45	0.313
813	3010-A	1	0.125	W	Y	5.95	0.04	0	0.187
814	3010-B	0	0.125	W	Y	7.12	0.04	0	0.187
815	3010-C	0	0.125	W	Y	7.45	0.04	0	0.187
816	3010-A1	0	0.104	W	N	7.29	0.04	0	0.25
817	3010-B1	0	0.125	W	Y	6.81	0.04	0	0.25
818	3007-A	0	0.03	W	N	6.58	0.032	65	0.187
819	3007-B	0	0.052	W	N	4.84	0.032	65	0.187
821	3008-A	0	0.076	W	N	4.67	0.032	65	0.25
822	3008-B	0	0.125	W	Y	4.27	0.032	65	0.25
824	3009-B	0	0.125	W	C	4.57	0.032	65	0.313
825	3011-B	1	0.125	W	Y	6.83	0.04	0	0.313
826	3011-A	3	0.125	W	Y	7.07	0.04	0	0.313
827	3012-B	1	0.125	W	Y	2.66	0.04	30	0.187
828	3012-D	0	0.112	W	N	4.32	0.04	30	0.187
829	3012-C	0	0.095	W	N	3.99	0.04	30	0.187
830	3013-A	0	0.125	W	Y	7.04	0.04	30	0.25
834	3020-B	0	0.125	W	Y	7.1	0.063	0	0.313
841	3024-B	0	0.125	W	Y	7.01	0.063	30	0.313
843	3022-D	1	0.125	W	Y	4.9	0.063	30	0.187
844	3020-A	0	0.125	W	Y	6.7	0.063	0	0.313
846	3028-A	1	0.125	W	Y	7.08	0.063	45	0.313
854	3034-A	0	0.125	W	Y	4.45	0.08	0	0.313
855	3034-B	0	0.125	W	Y	3.65	0.08	0	0.313
856	3034-C	0	0.125	W	Y	5.62	0.08	0	0.313
857	3036-A	1	0.125	W	Y	5.57	0.08	45	0.25
859	N2PURGE1	1	0.125	W	Y	6.88	0.063	0	0.313
861	3037-A	0	0.125	W	Y	7.09	0.08	45	0.313
863	3035-A	0	0.125	W	Y	5.29	0.08	45	0.187
865	3040-A	0	0.125	W	Y	6.91	0.08	65	0.313

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
866	3039-A	0	0.125	W	Y	5.65	0.08	65	0.25
868	3039-C	1	0.125	W	Y	6.27	0.08	65	0.25
931	2004-C	3	0.125	N	Y	6.07	0.063	0	0.25
939	2004-B	1	0.125	N	Y	6.93	0.063	0	0.25
941	2004-A	1	0.125	N	Y	7.23	0.063	0	0.25
963	3203	0	0.125	W	Y	6.09	0.032	45	0.187
964	3204	0	0.125	W	C	7.69	0.032	45	0.187
965	3201	0	0.125	W	Y	6.62	0.04	0	0.187
966	3202	0	0.003	W	N	7.19	0.04	0	0.187
967	3205	0	0.034	W	N	6.76	0.04	0	0.25
969	3206	0	0.125	W	Y	6.79	0.04	0	0.25
970	3207	2	0.125	W	Y	6.51	0.032	45	0.25
971	3208	0	0.017	W	N	3.8	0.032	65	0.25
972	3209	0	0.058	W	N	5.82	0.04	65	0.25
973	3211	0	0.027	W	N	5.67	0.04	65	0.25
974	3210	0	0.045	W	N	5.6	0.032	65	0.25
975	3212	0	0.125	W	Y	3.4	0.04	65	0.313
976	3213	0	0.094	W	N	4.44	0.04	65	0.313
977	3214	0	0.068	W	N	5.16	0.04	65	0.313
978	3215	0	0.105	W	N	5.65	0.04	65	0.313
979	3217	0	0.125	W	C	6.05	0.032	65	0.313
980	3216	0	0.125	W	S	6.3	0.04	65	0.313
981	3218	0	0.056	W	N	6.52	0.04	65	0.313
982	3212-1	1	0.125	W	Y	3.58	0.032	65	0.313
983	3209-1	0	0.06	W	N	4.55	0.04	65	0.25
997	3227-A	3	0.125	W	Y	6.8	0.063	0	0.375
1000	3227-D	3	0.125	B	Y	6.89	0.063	0	0.375
1002	3227-B	3	0.125	0.9	Y	6.64	0.063	0	0.375
1005	3301-A	0	0.125	B	Y	3.88	0.08	0	0.25
1006	3301-B	3	0.125	B	Y	4.26	0.08	0	0.25
1007	3301-C	0	0.094	B	N	6.26	0.08	0	0.25
1008	3301-D	0	0.12	B	N	5.47	0.08	0	0.25
1010	3302-A	1	0.125	B	Y	6.21	0.08	0	0.313
1011	3302-B	1	0.125	B	Y	6.42	0.08	0	0.313
1012	3302-C	0	0.125	B	Y	6.66	0.08	0	0.313
1015	3303-B	0	0.062	3.75	N	6.76	0.08	45	0.187
1016	3303-C	0	0.07	3.75	N	6.13	0.08	45	0.187
1017	3304-A	0	0.125	3.75	Y	6.12	0.08	45	0.25
1018	3304-B	1	0.125	3.75	Y	6.2	0.08	45	0.25
1019	3304-C	1	0.125	3.75	Y	6.85	0.08	45	0.25
1020	3304-D	0	0.125	3.75	Y	6.86	0.08	45	0.25
1021	3303-D	0	0.125	3.75	Y	5.68	0.08	45	0.187
1023	3305-B	0	0.083	3.75	N	4.17	0.063	0	0.187
1024	3305-C	0	0.125	3.75	C	3.81	0.063	0	0.187
1025	3305-A	0	0.125	3.75	N	4.63	0.063	0	0.187
1026	3306-A	0	0.085	3.75	N	5.23	0.063	0	0.25
1027	3306-C	0	0.125	3.75	Y	7.05	0.063	0	0.25
1028	3306-D	0	0.125	3.75	Y	6.98	0.063	0	0.25
1029	3301-E	0	0.125	3.75	Y	4.98	0.08	0	0.25
1030	3302-E	2	0.125	3.75	Y	6.22	0.08	0	0.313
1031	3306-B	0	0.092	3.75	N	6.98	0.063	0	0.25
1032	3307-A	0	0.065	3.75	N	4.77	0.063	45	0.187
1033	3307-B	0	0.033	3.75	N	6.27	0.063	45	0.187
1034	3308-A	0	0.125	3.75	Y	5.16	0.063	45	0.25

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
1035	3308-B	1	0.125	3.75	Y	6.2	0.063	45	0.25
1040	4001-B	2	0.125	3.75	Y	4.25	0.08	45	0.313
1041	4001-C	2	0.125	3.75	Y	6.11	0.08	45	0.313
1042	4001-A	3	0.125	3.75	Y	3.18	0.08	45	0.313
1043	4001-D	1	0.125	3.75	Y	6.71	0.08	45	0.313
1044	4002-B	0	0.035	3.75	N	3.98	0.08	75	0.313
1045	4002-C	0	0.125	3.75	Y	6.29	0.08	75	0.313
1046	4002-A	0	0.01	3.75	N	3.2	0.08	75	0.313
1047	4002-D	0	0.065	3.75	N	7.13	0.063	75	0.313
1048	4003-A	3	0.125	3.75	Y	3.42	0.08	45	0.313
1049	4003-B	2	0.125	3.75	Y	6.28	0.08	45	0.313
1050	4003-D	1	0.125	3.75	Y	6.24	0.08	45	0.313
1051	4003-C	3	0.125	3.75	Y	3.18	0.08	45	0.313
1052	4004-A	0	0.016	3.75	N	3.19	0.08	75	0.313
1053	4004-B	0	0.125	3.75	Y	6.07	0.08	75	0.313
1054	4004-C	0	0.125	3.75	C	6.19	0.08	75	0.313
1056	4002-E	0	0.076	3.75	N	6.36	0.08	75	0.313
1058	4109-A	0	0.042	3.75	N	3.27	0.08	45	0.187
1059	4109-B	0	0.048	3.75	N	4.14	0.08	45	0.187
1060	4109-C	0	0.041	3.75	N	6.53	0.08	45	0.187
1062	4109-D	0	0.032	3.75	N	7.44	0.08	45	0.187
1064	4110-A	0	0.081	3.75	N	3.26	0.08	45	0.25
1065	4110-B	0	0.115	3.75	N	3.99	0.08	45	0.25
1068	4110-C	0	0.125	3.75	Y	5.77	0.08	45	0.25
1069	4110-D	0	0.057	3.75	N	6.91	0.08	45	0.25
1070	4111-A	0	0.125	3.75	Y	2.85	0.08	45	0.313
1071	4111-B	2	0.125	3.75	Y	3.94	0.08	45	0.313
1072	4111-C	3	0.125	3.75	Y	5.97	0.08	45	0.313
1073	4111-D	0	0.125	3.75	Y	6.81	0.08	45	0.313
1074	4112-A	0	0.021	3.75	N	3.34	0.08	60	0.187
1075	4112-B	0	0.03	3.75	N	4.03	0.08	60	0.187
1076	4112-C	0	0.036	3.75	N	5.89	0.08	60	0.187
1077	4112-D	0	0.049	3.75	N	7.51	0.08	60	0.187
1078	4113-A	0	0.007	3.75	N	2.97	0.08	60	0.25
1079	4113-B	0	0.05	3.75	N	3.78	0.08	60	0.25
1080	4113-C	0	0.125	3.75	Y	6.31	0.08	60	0.25
1082	4113-D	0	0.084	3.75	N	7.13	0.08	60	0.25
1083	4114-A	0	0.024	3.75	N	3.14	0.08	60	0.313
1084	4114-B	0	0.106	3.75	N	3.98	0.08	60	0.313
1085	4114-C	0	0.125	3.75	Y	5.93	0.08	60	0.313
1086	4114-D	0	0.125	3.75	Y	7.42	0.08	60	0.313
1088	4115-B	0	0	3.75	N	4.08	0.08	75	0.187
1089	4115-A	0	0	3.75	N	3.14	0.08	75	0.187
1090	4115-C	0	0.017	3.75	N	6.06	0.08	75	0.187
1091	4115-D	0	0.019	3.75	N	7.28	0.08	75	0.187
1092	4116-A	0	0	3.75	N	2.93	0.08	75	0.25
1093	4116-B	0	0	3.75	N	4.47	0.08	75	0.25
1094	4116-C	0	0.034	3.75	N	6.21	0.08	75	0.25
1095	4116-D	0	0.025	3.75	N	7.35	0.08	75	0.25
1096	4117-A	0	0.001	3.75	N	3.12	0.08	75	0.313
1097	4117-B	0	0.125	3.75	Y	4.06	0.08	75	0.313
1098	4117-C	0	0.108	3.75	N	6.01	0.08	75	0.313
1103	4117-D	0	0.078	3.75	N	7.11	0.08	75	0.313
1105	4100-A	0	0.056	3.75	N	3	0.05	45	0.187

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
1106	4100-B	0	0.052	3.75	N	3.79	0.05	45	0.187
1108	4100-C	0	0.103	3.75	N	5.62	0.05	45	0.187
1109	4100-D	0	0.047	3.75	N	7.15	0.05	45	0.187
1110	4101-A	1	0.125	3.75	Y	3.15	0.05	45	0.25
1111	4101-B	1	0.125	3.75	Y	4.14	0.05	45	0.25
1112	4101-C	2	0.125	3.75	Y	6.12	0.05	45	0.25
1113	4101-D	0	0.125	3.75	Y	7.48	0.05	45	0.25
1114	4103-A	0	0.001	3.75	N	2.93	0.05	60	0.187
1115	4103-B	0	0.019	3.75	N	4.01	0.05	60	0.187
1116	4103-C	0	0.024	3.75	N	5.9	0.05	60	0.187
1118	4104-A	0	0.043	3.75	N	3.19	0.05	60	0.25
1121	4103-D	0	0.027	3.75	N	7.37	0.05	60	0.187
1122	4104-B	0	0.052	3.75	N	4.19	0.05	60	0.25
1123	4104-C	0	0.041	3.75	N	6.13	0.05	60	0.25
1124	4104-D	0	0.125	3.75	N	7.55	0.05	60	0.25
1125	4106-A	0	0.001	3.75	N	3.03	0.05	60	0.187
1126	4106-B	0	0.027	3.75	N	4.13	0.05	60	0.187
1128	4107-A	0	0	3.75	N	3.06	0.05	75	0.25
1130	4107-B	0	0.001	3.75	N	4.13	0.05	75	0.25
1131	4107-C	0	0.025	3.75	N	6.23	0.05	75	0.25
1132	4107-D	0	0.064	3.75	N	7.66	0.05	75	0.25
1133	4106-A1	0	0	3.75	N	3.11	0.05	75	0.187
1134	4106-B1	0	0.001	3.75	N	4.03	0.05	75	0.187
1136	4106-D	0	0.001	3.75	N	7.56	0.05	75	0.187
1137	4106-C	0	0.001	3.75	N	5.97	0.05	75	0.187
1138	3401-D	1	0.125	2.0	Y	5.27	0.063	0	0.313
1139	3401-C	0	0.125	2.0	Y	6.15	0.063	0	0.313
1140	3401-A	3	0.125	2.0	Y	7.23	0.063	0	0.313
1141	3402-D	1	0.125	2.0	Y	5.04	0.063	0	0.313
1142	3402-C	0	0.125	2.0	Y	6.19	0.063	0	0.313
1143	3402-A	2	0.125	2.0	Y	7.21	0.063	0	0.313
1144	3401-B	2	0.125	2.0	Y	7.11	0.063	0	0.313
1145	3402-B	2	0.125	2.0	Y	7.13	0.063	0	0.313
1146	3403-D	2	0.125	3.75	Y	5.07	0.063	0	0.313
1147	3403-C	2	0.125	3.75	Y	6.22	0.063	0	0.313
1148	3403-B	2	0.125	3.75	Y	7.13	0.063	0	0.313
1149	3403-A	2	0.125	3.75	Y	7.17	0.063	0	0.313
1150	MLI-BURN2	2	0.125	3.75	Y	7.19	0.063	0	0.313
1152	3404-A	3	0.125	2.0	Y	6.85	0.063	0	0.375
1153	3404-B	3	0.125	2.0	Y	6.85	0.063	0	0.375
1154	3404-C	3	0.125	2.0	Y	6.23	0.063	0	0.375
1156	3404-D	2	0.125	2.0	Y	5.52	0.063	0	0.375
1157	3407-D	0	0.109	3.75	N	5.83	0.063	0	0.25
1158	3407-C	0	0.125	3.75	Y	4.81	0.063	0	0.25
1159	3407-B	3	0.125	3.75	Y	3.97	0.063	0	0.25
1160	3407-A	3	0.125	3.75	Y	2.92	0.063	0	0.25
1161	3406-A	1	0.125	2.0	Y	3.08	0.063	0	0.25
1162	3406-B	0	0.103	2.0	N	4.07	0.063	0	0.25
1163	3406-C	0	0.047	2.0	N	5.18	0.063	0	0.25
1164	3406-D	0	0.118	2.0	N	5.58	0.063	0	0.25
1165	3405-A	2	0.125	2.0	Y	3.06	0.063	0	0.25
1166	3405-B	1	0.125	2.0	Y	3.86	0.063	0	0.25
1169	3406-D1	0	0.125	2.0	Y	6.13	0.063	0	0.25
1170	9001-1	0	0.104	3.75	N	6.07	0.063	45	0.25

tnum	tname	witpen	rwpend	mli2	rearpen?	velocity	bumgage	oblique	prjsize
1171	9001-A	0	0.125	3.75	Y	6	0.05	45	0.25
1172	9001-B	2	0.125	3.75	Y	5.95	0.05	45	0.25
1173	9001-C	0	0.108	3.75	N	5.97	0.05	45	0.25
1174	9001-D	0	0.125	3.75	Y	6.1	0.05	45	0.25
1175	9002-A	2	0.125	3.75	Y	6.41	0.05	45	0.25
1176	9002-B	0	0.106	3.75	N	6.43	0.05	45	0.25
1177	9002-C	0	0.128	3.75	N	6.36	0.05	45	0.25
1178	9002-D	0	0.085	3.75	N	6.38	0.05	45	0.25
1179	4108-B	0	0.035	3.75	N	3.97	0.05	75	0.313
1180	4108-C	0	0.074	3.75	N	5.95	0.05	75	0.313
1182	4105-C	0	0.068	3.75	N	6.15	0.05	60	0.313
1183	4105-D	0	0.056	3.75	N	7.25	0.05	60	0.313
1184	4105-B	0	0.095	3.75	N	4.04	0.05	60	0.313
1185	4105-A	2	0.125	3.75	Y	2.93	0.05	60	0.313
1186	4108-A	0	0.001	3.75	N	3.13	0.05	75	0.313
1187	4105-A1	0	0.09	3.75	N	3.01	0.05	60	0.313
1188	4108-A1	0	0.001	3.75	N	2.96	0.05	75	0.313
1190	4102-A	2	0.125	3.75	Y	2.95	0.05	45	0.313

APPENDIX A.2 - 221 SHOT DATASET FOR FINAL REGRESSION

Final Dataset prepared to perform Generalized Regression

witpen	rwpend	velocity	bumgage	cos θ	projdia
6	0.3175				
0	0.3175	7.45	0.1016	1	0.47498
2	0.3175	7.39	0.08128	1	0.635
0	0.26416	7.29	0.1016	1	0.635
3	0.3175	7.23	0.16002	1	0.79502
2	0.3175	7.21	0.16002	1	0.79502
0	0.00762	7.19	0.1016	1	0.47498
2	0.3175	7.19	0.16002	1	0.79502
2	0.3175	7.17	0.16002	1	0.79502
2	0.3175	7.13	0.16002	1	0.79502
2	0.3175	7.13	0.16002	1	0.79502
0	0.3175	7.12	0.1016	1	0.47498
2	0.3175	7.11	0.16002	1	0.79502
0	0.3175	7.1	0.16002	1	0.79502
3	0.3175	7.07	0.1016	1	0.79502
0	0.3175	7.05	0.16002	1	0.635
0	0.23368	6.98	0.16002	1	0.635
0	0.3175	6.98	0.16002	1	0.635
3	0.3175	6.89	0.16002	1	0.9525
0	0.0254	6.89	0.16002	1	0.762
1	0.3175	6.88	0.16002	1	0.79502
3	0.3175	6.85	0.16002	1	0.9525
3	0.3175	6.85	0.16002	1	0.9525
1	0.3175	6.83	0.1016	1	0.79502
0	0.3175	6.82	0.08128	1	0.47498
0	0.3175	6.81	0.1016	1	0.635
3	0.3175	6.8	0.16002	1	0.9525
0	0.3175	6.79	0.1016	1	0.635
0	0.08636	6.76	0.1016	1	0.635
0	0.3175	6.7	0.16002	1	0.79502
0	0.3175	6.66	0.2032	1	0.79502
3	0.3175	6.64	0.16002	1	0.9525
0	0.3175	6.62	0.1016	1	0.47498
0	0.0254	6.6	0.16002	1	0.762
1	0.3175	6.42	0.2032	1	0.79502
0	0.3175	6.27	0.08128	1	0.47498
0	0.23876	6.26	0.2032	1	0.635
3	0.3175	6.23	0.16002	1	0.9525
2	0.3175	6.22	0.2032	1	0.79502
2	0.3175	6.22	0.16002	1	0.79502
1	0.3175	6.21	0.2032	1	0.79502
0	0.3175	6.19	0.16002	1	0.79502
6	0.3175	6.16	0.16002	1	0.66548
0	0.3175	6.15	0.16002	1	0.79502
0	0.3175	6.13	0.16002	1	0.635
2	0.3175	6.13	0.16002	1	0.79502
0	0.0254	5.96	0.2032	1	0.762
1	0.3175	5.95	0.1016	1	0.47498
1	0.3175	5.85	0.16002	1	0.762
0	0.27686	5.83	0.16002	1	0.635
1	0.3175	5.78	0.08128	1	0.47498
0	0.3175	5.63	0.16002	1	0.79502
0	0.3175	5.62	0.2032	1	0.79502
0	0.29972	5.58	0.16002	1	0.635
2	0.3175	5.52	0.16002	1	0.9525
0	0.3048	5.47	0.2032	1	0.635
0	0.0508	5.35	0.2032	1	0.762
1	0.3175	5.27	0.16002	1	0.79502
0	0.2159	5.23	0.16002	1	0.635

0	0.11938	5.18	0.16002	1	0.635
2	0.3175	5.07	0.16002	1	0.79502
1	0.3175	5.04	0.16002	1	0.79502
0	0.3175	4.98	0.2032	1	0.635
1	0.3175	4.85	0.1016	1	0.635
0	0.3175	4.81	0.16002	1	0.635
0	0.3175	4.74	0.2032	1	0.762
0	0.3175	4.63	0.16002	1	0.47498
0	0.3175	4.45	0.2032	1	0.79502
0	0.254	4.33	0.16002	1	0.635
3	0.3175	4.26	0.2032	1	0.635
0	0.21082	4.17	0.16002	1	0.47498
0	0.26162	4.07	0.16002	1	0.635
2	0.3175	3.99	0.08128	1	0.47498
3	0.3175	3.97	0.16002	1	0.635
0	0.3048	3.96	0.16002	1	0.635
0	0.3175	3.88	0.2032	1	0.635
1	0.3175	3.86	0.16002	1	0.635
0	0.3175	3.83	0.2032	1	0.762
0	0.3175	3.81	0.16002	1	0.47498
0	0.1524	3.68	0.16002	1	0.47498
0	0.3175	3.65	0.2032	1	0.79502
0	0.0762	3.08	0.16002	1	0.47498
1	0.3175	3.08	0.16002	1	0.635
2	0.3175	3.06	0.16002	1	0.635
1	0.3175	2.96	0.16002	1	0.635
0	0.3175	2.93	0.16002	1	0.635
3	0.3175	2.92	0.16002	1	0.635
0	0.127	2.83	0.16002	1	0.47498
0	0.3048	2.54	0.16002	1	0.47498
0	0.3175	2.45	0.16002	1	0.47498
3	0.3175	6.49	0.1016	0.707107	0.9525
4	0.3175	6.97	0.16002	0.707107	0.889
4	0.3175	5.87	0.16002	0.707107	0.889
3	0.3175	7.12	0.08128	0.707107	0.79502
0	0.3175	7.09	0.2032	0.707107	0.79502
1	0.3175	7.08	0.16002	0.707107	0.79502
2	0.3175	7.02	0.1016	0.707107	0.79502
4	0.3175	6.9	0.1016	0.707107	0.79502
0	0.3175	6.81	0.2032	0.707107	0.79502
1	0.3175	6.71	0.2032	0.707107	0.79502
4	0.3175	6.54	0.1016	0.707107	0.79502
0	0.3175	6.53	0.2032	0.707107	0.79502
0	0.3175	6.45	0.16002	0.707107	0.79502
2	0.3175	6.28	0.2032	0.707107	0.79502
1	0.3175	6.24	0.2032	0.707107	0.79502
2	0.3175	6.11	0.2032	0.707107	0.79502
3	0.3175	5.97	0.2032	0.707107	0.79502
1	0.3175	4.25	0.16002	0.707107	0.79502
3	0.3175	4.25	0.2032	0.707107	0.79502
2	0.3175	4.25	0.2032	0.707107	0.79502
4	0.3175	4.12	0.1016	0.707107	0.79502
2	0.3175	3.94	0.2032	0.707107	0.79502
2	0.3175	3.72	0.16002	0.707107	0.79502
3	0.3175	3.42	0.2032	0.707107	0.79502
3	0.3175	3.18	0.2032	0.707107	0.79502
3	0.3175	3.18	0.2032	0.707107	0.79502
0	0.3175	3.08	0.16002	0.707107	0.79502
2	0.3175	3.01	0.2032	0.707107	0.79502
4	0.3175	2.99	0.1016	0.707107	0.79502
2	0.3175	2.95	0.127	0.707107	0.79502
0	0.3175	2.85	0.2032	0.707107	0.79502
2	0.3175	6.27	0.16002	0.707107	0.762
1	0.3175	5.57	0.2032	0.707107	0.635

0	0.3175	6.86	0.2032	0.707107	0.635
1	0.3175	6.2	0.2032	0.707107	0.635
1	0.3175	6.85	0.2032	0.707107	0.635
0	0.3175	6.12	0.2032	0.707107	0.635
0	0.14478	6.91	0.2032	0.707107	0.635
0	0.20574	3.26	0.2032	0.707107	0.635
0	0.3175	5.77	0.2032	0.707107	0.635
0	0.2921	3.99	0.2032	0.707107	0.635
0	0.3175	4.61	0.16002	0.707107	0.635
0	0.3175	4.16	0.16002	0.707107	0.635
1	0.3175	5.3	0.16002	0.707107	0.635
0	0.3175	6.3	0.16002	0.707107	0.635
2	0.3175	3.15	0.16002	0.707107	0.635
0	0.26416	6.07	0.16002	0.707107	0.635
1	0.3175	6.2	0.16002	0.707107	0.635
0	0.3175	5.16	0.16002	0.707107	0.635
0	0.27432	5.97	0.127	0.707107	0.635
1	0.3175	4.14	0.127	0.707107	0.635
2	0.3175	5.95	0.127	0.707107	0.635
1	0.3175	3.15	0.127	0.707107	0.635
0	0.32512	6.36	0.127	0.707107	0.635
2	0.3175	6.41	0.127	0.707107	0.635
0	0.3175	7.48	0.127	0.707107	0.635
0	0.26924	6.43	0.127	0.707107	0.635
0	0.2159	6.38	0.127	0.707107	0.635
0	0.3175	6	0.127	0.707107	0.635
2	0.3175	6.12	0.127	0.707107	0.635
0	0.3175	6.1	0.127	0.707107	0.635
0	0.3175	5.76	0.1016	0.707107	0.635
0	0.3175	7.21	0.1016	0.707107	0.635
1	0.3175	4.12	0.1016	0.707107	0.635
4	0.3175	5.51	0.1016	0.707107	0.635
1	0.3175	4.33	0.1016	0.707107	0.635
2	0.3175	4.54	0.1016	0.707107	0.635
2	0.3175	7.59	0.1016	0.707107	0.635
2	0.3175	6.51	0.08128	0.707107	0.635
0	0.3175	7.46	0.08128	0.707107	0.635
0	0.23876	7.69	0.08128	0.707107	0.635
0	0.3175	7.69	0.08128	0.707107	0.47498
0	0.08128	7.44	0.2032	0.707107	0.47498
0	0.11938	7.15	0.127	0.707107	0.47498
0	0.15748	6.76	0.2032	0.707107	0.47498
0	0.3175	6.64	0.08128	0.707107	0.47498
0	0.10414	6.53	0.2032	0.707107	0.47498
0	0.11684	6.42	0.1016	0.707107	0.47498
0	0.08382	6.27	0.16002	0.707107	0.47498
0	0.1778	6.13	0.2032	0.707107	0.47498
0	0.3175	6.09	0.08128	0.707107	0.47498
0	0.0762	5.93	0.1016	0.707107	0.47498
0	0.3175	5.68	0.2032	0.707107	0.47498
0	0.26162	5.62	0.127	0.707107	0.47498
0	0.3175	5.29	0.2032	0.707107	0.47498
0	0.1651	4.77	0.16002	0.707107	0.47498
0	0.21336	4.6	0.1016	0.707107	0.47498
0	0.17272	4.41	0.16002	0.707107	0.47498
0	0.12192	4.14	0.2032	0.707107	0.47498
0	0.3175	4.08	0.1016	0.707107	0.47498
0	0.13208	3.79	0.127	0.707107	0.47498
0	0.27686	3.66	0.1016	0.707107	0.47498
0	0.10668	3.27	0.2032	0.707107	0.47498
0	0.1016	3.23	0.16002	0.707107	0.47498
0	0.14224	3	0.127	0.707107	0.47498
0	0.24892	2.93	0.1016	0.707107	0.47498
0	0.25908	7.34	0.16002	0.422618	0.635

0	0.12446	7.03	0.16002	0.422618	0.762
1	0.3175	6.93	0.16002	0.422618	0.889
0	0.3175	6.91	0.2032	0.422618	0.79502
0	0.19558	6.72	0.1016	0.422618	0.762
0	0.0762	6.58	0.08128	0.422618	0.47498
0	0.14224	6.52	0.1016	0.422618	0.79502
0	0.18796	6.35	0.16002	0.422618	0.635
0	0.3175	6.3	0.1016	0.422618	0.79502
1	0.3175	6.27	0.2032	0.422618	0.635
0	0.3175	6.25	0.16002	0.422618	0.762
0	0.3175	6.05	0.08128	0.422618	0.79502
0	0.14732	5.82	0.1016	0.422618	0.635
0	0.3175	5.74	0.16002	0.422618	0.762
0	0.3175	5.69	0.16002	0.422618	0.889
0	0.06858	5.67	0.1016	0.422618	0.635
0	0.3175	5.65	0.2032	0.422618	0.635
0	0.2667	5.65	0.1016	0.422618	0.79502
0	0.1143	5.6	0.08128	0.422618	0.635
0	0.10414	5.59	0.1016	0.422618	0.762
0	0.17272	5.16	0.1016	0.422618	0.79502
0	0.13208	4.84	0.08128	0.422618	0.47498
0	0.14224	4.79	0.1016	0.422618	0.762
0	0.19304	4.67	0.08128	0.422618	0.635
0	0.3175	4.64	0.1016	0.422618	0.889
0	0.3175	4.57	0.08128	0.422618	0.79502
0	0.1524	4.55	0.1016	0.422618	0.635
0	0.23876	4.44	0.1016	0.422618	0.79502
0	0.18288	4.29	0.16002	0.422618	0.635
0	0.3175	4.27	0.08128	0.422618	0.635
0	0.04318	3.8	0.08128	0.422618	0.635
0	0.32004	3.65	0.1016	0.422618	0.762
1	0.3175	3.58	0.08128	0.422618	0.79502
0	0.3175	3.4	0.1016	0.422618	0.79502
0	0.3175	3.05	0.1016	0.422618	0.889
0	0.1651	2.72	0.1016	0.422618	0.762

APPENDIX A.3 - DISCARDED SHOTS

Executing the HITS database program will produce 396 shots when set-up for dual wall targets similar to Space Station Freedom's protection system. This will reduce to 385 when the following shot numbers are removed for the given reasons.

Shot Number	Reason for Discarding
52	No penetration depth data provided and hardware not available.
53	No penetration depth data provided and hardware not available.
54	No penetration depth data provided and hardware not available.
57	No penetration depth data provided and hardware not available.
163	No penetration depth data provided and hardware not available.
179	No penetration depth data provided and hardware not available.
424	MLI position indicated by "CPR" - Designation unknown.
426	MLI position indicated by "CPR" - Designation unknown.
721	No penetration depth data provided and hardware not available.
1167	Shot sheet indicates multiple holes in Bumper - Premature Fragmentation.
1168	Shot sheet indicates multiple holes in Bumper - Premature Fragmentation.

APPENDIX B

REGRESSION AND STATISTICAL ANALYSIS FORTRAN COMPUTER CODES

APPENDIX B.1 - MDREG.FOR, SINGLE REGRESSION CODE WITH ANOVA

```
C
C      THIS IS A REGRESSION PROGRAM CALLED MDREG.FOR
C
C      WRITTEN BY WILLIAM H. JOLLY
C         Last Revision on:                        May 4, 1992
C
      PARAMETER (M=500,N=8)
C      M IS THE MAXIMUM SIZE OF THE ARRAYS
C        CORRESPONDING TO THE NUMBER OF DATA POINTS.
C      N IS THE MAXIMUM SIZE OF THE ARRAYS
C        CORRESPONDING TO THE NUMBER OF INDEPENDENT VARIABLES.
C      T IS THE t STATISTIC t(v,1-a/2) = 1.98 FOR INFINITE DF
C        AND 95% PROBABILITY.
      DIMENSION RD(M,N),X(M,N),C[ALLOCATABLE](:,:),SV[ALLOCATABLE](:),
     1  Z[ALLOCATABLE](:),X0[ALLOCATABLE](:),XIN[ALLOCATABLE](:,:),
     1  D[ALLOCATABLE](:)
      REAL MSREG,MSRES,ZTOL
      INTEGER NER, DOPRT, DF, ERROR
      CHARACTER *20 INFILE,DATFILE,OUTFILE,LSTFILE,ANS,RPLT,ANOV
      DATA DOPRT/1/
      PRINT*,'NON-LINEAR REGRESSION OF A BALLISTIC LIMIT EQUATION:'
      PRINT*,'                        c1  c2   c3      c4     c5'
      PRINT*,'      P* = P+1 = e   *V   *Tb   *cosO   *DIA'
      PRINT*,' '
      PRINT*,'THE REDUCED BALLISTIC EQUATION IS:'
      PRINT*,'                      z2    z3      z4'
      PRINT*,'      DIA = z1*V   *Tb   *cosO'
      PRINT*,' '
      PRINT*,'REQUIRED INPUT DATASET FORMAT:'
      PRINT*,'* FIRST LINE IN THE INPUT DATASET MUST CONTAIN AN INTEGER'
      PRINT*,'  INDICATING THE NUMBER OF COLUMNS IN THE DATASET AND THE'
      PRINT*,'  REAR WALL THICKNESS'
      PRINT*,'* THE DATASET FOLLOWS.'
      PRINT*,'    THIS PROGRAM INSERTS A COLUMN OF ONES AT THE FIRST'
      PRINT*,'    POSITION TO REPRESENT THE Y INTERCEPT FOR A LINEAR'
      PRINT*,'    REGRESSION.  THE REST OF THE COLUMNS ARE OPERATED ON'
      PRINT*,'    BY TAKING THE LOG TO MAP A NON-LINEAR FIT INTO A'
      PRINT*,'    LINEAR REGRESSION MODEL.  THE LAST COLUMN MUST'
      PRINT*,'    CONTAIN THE DEPENDENT VARIABLE.'
      PRINT*,' '
 30   PRINT*,'DO YOU WANT AN OUTPUT FILE (y/n)?'
      PRINT*,'ENTER x TO EXIT.'
      READ(*,1,ERR=30) ANS
      IF(ANS.EQ.'x') STOP
      IF(ANS.EQ.'n') GOTO 40
      IF(ANS.NE.'y') GOTO 30
 40   PRINT*,' '
      PRINT*,'ENTER THE INPUT DATA FILENAME, NO EXTENSION:'
      PRINT*,'   THE EXTENSION OF .dat IS ASSUMMED.'
      PRINT*,'ENTER x TO EXIT.'
      READ(*,1,ERR=40) INFILE
      IF(INFILE.EQ.'x') STOP
      L=LEN_TRIM(INFILE)
      DATFILE = INFILE(:L)//'.dat'
      PRINT*,' '
      PRINT*,'INPUT FILENAME IS: ',DATFILE
      OPEN(UNIT=5,FILE=DATFILE,STATUS='OLD',ERR=35)
      GOTO 42
 35   PRINT*,' '
      PRINT*,'File NOT found.'
      PRINT*,'Enter correct path or filename.'
      GOTO 40
 42   IF(ANS.EQ.'n') GOTO 43
      OUTFILE = INFILE(:L)//'.out'
```

```
      PRINT*,' '
      PRINT*,'THE OUTPUT FILENAME IS: ',OUTFILE
      OPEN(UNIT=6,FILE=OUTFILE)
   43 PRINT*,' '
      PRINT*,'DO YOU WANT TO PLOT RESIDUALS VS. VARIABLES? (y/n)'
      PRINT*,'ACTUAL, PREDICTED, RESIDUAL, AND INPUT VARIABLES'
      READ(*,1,ERR=43) RPLT
      IF(RPLT.EQ.'n') GOTO 45
      IF(RPLT.NE.'y') GOTO 43
      DOPRT = 0
      LSTFILE = INFILE(:L)//'.lst'
      PRINT*,' '
      PRINT*,'THE LISTING FILENAME IS: ',LSTFILE
      OPEN(UNIT=7,FILE=LSTFILE)
C   ******************************************************************
C       THIS IS THE ALTERABLE CODE SECTION FOR THE REGRESSION
C       ROUTINE.  READ THE DATA, OPERATE ON IT, AND LOAD IT INTO
C       THE X MATRIX.  THEN CALL RSTAT.
C   ******************************************************************
   45 READ(5,*) NC, RWT
      IF(NC.LE.N)GOTO 48
        PRINT*,'TOO MANY COLUMNS !!  MUST NOT BE GREATER THAN',N
        STOP
   48 RWTHK=RWT*.9999
      NX = NC - 1
      NT = NC - 2
      NC1= NC + 1
      ALLOCATE (C(NC,NC),STAT=ERROR)
      ALLOCATE (XIN(NX,NX),STAT=ERROR)
      ALLOCATE (SV(NX),STAT=ERROR)
      ALLOCATE (Z(NT),STAT=ERROR)
      ALLOCATE (X0(NX),STAT=ERROR)
      ALLOCATE (D(NX),STAT=ERROR)
C      NC IS NUMBER OF COLUMNS IN THE INPUT DATASET
C      NX IS THE NUMBER OF TERMS ON RHS.  INCLUCES A0.
C      NT IS THE NUMBER OF INDEPENDENT VARIABLES. X1, X2, X3, ...
C      NP IS THE NUMBER OF DATA POINTS IN THE INPUT FILE.
C      RWTHK IS THE REAR WALL THICKNESS.  THE VALUE IS REDUCED TO ASSURE
C      INTEGER ROUNDUP TO ONE IN THE PENETRATION PARAMETER EQUATION.
      I=1
   50 READ(5,*,END=55)(RD(I,J),J=1,NC)
      I=I+1
      GOTO 50
   55 NP=I-1
      DF = NP-NT-1
      IF(NP.GT.NC) GOTO 60
      PRINT*,'NOT ENOUGH DATA POINTS TO PERFORM A REGRESSION.'
      PRINT*,'TRY A CURVE FITTING ROUTINE.'
   60 DO 75 I=1,NP
      RD(I,NC1)=RD(I,2)*2.851+(RD(I,1)+0.5*INT(RD(I,2)/RWTHK))
     1                *0.14202+1.0
      X(I,1)=1.0
      DO 70 J=2,NC
   70   X(I,J)=ALOG(RD(I,J+1))
   75 CONTINUE
      CALL NORM(X,NP,NC,M,SYY,YBAR,C,SV)
C      X0 IS THE VECTOR FOR WHICH THE CONFIDENCE INTERVAL
C        IS DETERMINED (MEAN OR AVERAGE VALUES)
      DO 90 I=1,NX
   90   X0(I)=C(I,1)/C(1,1)
      CALL CHOL(C,NC,NER)
      IF(NER.EQ.0) GOTO 85
      WRITE(*,994)
      WRITE(6,994)
```

```
      STOP
   85 CALL FSOLR(C,SV,NX,NC)
      CALL BSOLRT(C,SV,NX,NC)
      CALL STAT(X,NP,NX,M,SV,DOPRT,SE,COR,FSTAT,SYY,YBAR,SSTC,SSREG)
      MSREG=SSREG/NT
      SSRES=SSTC-SSREG
      MSRES=SSRES/DF
C      DETERMINE CONFIDENCE INTERVALS
C      BEGIN BY DETERMINING THE INVERSE OF THE NORMAL MATRIX
      CALL CINV(C,XIN,NX,NC;D)
C      PX0 IS THE PREDICTED VALUE OF P* FOR X0 - CENTER FOR C.I.
      PX0=SV(1)
      DO 95 I=2,NX
   95   PX0=PX0+X0(I)*SV(I)
      CALL CI(XIN,X0,NX,SE,DEL)
      TSTAT=1.96
      IF(NP.GT.110) GOTO 96
        WRITE(*,5) NP
    5 FORMAT(1H ,//' Not enough data points to justify using the',
     1 ' infinite value for the t statistic.',
     1 /' ENTER T(',I3,',0.0975) [default=1.96]')
        READ(*,2,ERR=96) TSTAT
   96 TOL=TSTAT*DEL
      PMIN=EXP(PX0-TOL)
      PMAX=EXP(PX0+TOL)
      PX0=EXP(PX0)
C      CALCULATE EXPONENTS IN NONLINEAR EQN SOLVED FOR DIA
      PC=(RWT*2.851+1.0)
      Z(1)=(PC/EXP(SV(1)))**(1/SV(NX))
      DO 100 J=2,NT
  100   Z(J)=-SV(J)/SV(NX)
      ZTOL=EXP(TOL/SV(NX))
C   *****************************************************************
C      WELCOME TO THE OUTPUT SECTION
C   *****************************************************************
      WRITE(*,995) DATFILE
      IF(NER.EQ.0) GOTO 900
      WRITE(*,1100) NER
  900 WRITE(*,1102) NP,NT,DF
      WRITE(*,1115)
      WRITE(*,1105)(J,SV(J),J=1,NX)
      WRITE(*,1130) PC
      WRITE(*,1125)(J,Z(J),J=1,NT)
  905 PRINT*,'Do you want to see Statistics? (y/n)'
      READ(*,1) ANOV
      IF(ANOV.EQ.'n') GOTO 910
      IF(ANOV.NE.'y') GOTO 905
      WRITE(*,1110) NT,SSREG,MSREG,FSTAT,DF,SSRES,MSRES,NP-1,SSTC,
     1              COR,SE
  910 IF(ANS.EQ.'n') GOTO 990
      WRITE(6,995) DATFILE
      IF(NER.EQ.0) GOTO 920
      WRITE(6,1100) NER
  920 WRITE(6,1102) NP,NT,DF
      WRITE(6,1115)
      WRITE(6,1105)(J,SV(J),J=1,NX)
      WRITE(6,1130) PC
      WRITE(6,1125)(J,Z(J),J=1,NT)
      WRITE(6,1110) NT,SSREG,MSREG,FSTAT,DF,SSRES,MSRES,NP-1,SSTC,
     1              COR,SE
      WRITE(6,1114)
      WRITE(6,1113)(EXP(X0(J)),J=2,NX)
      WRITE(6,1116) PX0,PMIN,PMAX,TSTAT,ZTOL
  990 PRINT*,' '
```

```
      PRINT*,'DO YOU HAVE ANOTHER INPUT FILE (y/n)?'
      READ(*,1,ERR=990) ANS
      IF(ANS.EQ.'n') STOP
      IF(ANS.NE.'y') GOTO 990
      CLOSE(UNIT=5)
      GOTO 30
  1   FORMAT(A20)
  2   FORMAT(F12.4)
  994 FORMAT(1H ,/' The normal matrix is SINGULAR.')
  995 FORMAT(1H ,'    MODEL: mdreg.for     INPUT FILENAME: ',A20)
 1100 FORMAT(1H ,'**MATRIX SOLUTION ERROR, CONVERGENCE NOT COMPLETE OR',
     1          /'  MATRIX IS SINGULAR.   ERROR INDICATOR = ',I3)
 1102 FORMAT(/'     NON-LINEAR REGRESSION ANALYSIS for',I5,
     1           ' observations',/'     using',I5,'   variables',
     1           ' resulting in',I5,' Degrees of Freedom:')
 1105 FORMAT(1H ,T30,'C(',I1,')=',E14.5)
 1110 FORMAT(1H ,/' ',T32,'ANOVA',
     1/'     ---------------------------------------------------',
     1 '-------',
     1/'     Source',T21,'df',T34,'SS',T49,'MS',T60,'F Value',
     1/'     Regression',T20,I3,T25,E14.5,T40,E14.5,T55,E14.5,
     1/'     Residual',T20,I3,T25,E14.5,T40,E14.5,
     1/'     ---------------------------------------------------',
     1 '-------',
     1/'     Total'
     1/'     Corrected',T20,I3,T25,E14.5,
     1//'     Multiple Correlation Coefficient:',T41,'(R2)=',T55,E14.5
     1/'     Estimate of Variance:',T41,'(SE2)=',T55,E14.5)
 1113 FORMAT(1H ,T10,E14.5,T25,E14.5,T40,E14.5,T55,E14.5)
 1114 FORMAT(1H ,/'     Mean Vector (Xo):',/' ',T14,'Velocity',T33,
     1 'Tb',T48,'cosO',T63,'Dia')
 1116 FORMAT(1H ,/'     Confidence Interval:',
     1/' ',T10,'Predicted Mean Solution:',T41,'P*(Xo)=',T55,F7.3,
     1/' ',T10,'95% Confidence Interval on P* is:   [',
     1             F7.3,',',F7.3,']',
     1/' ',T10,'The specified value of the t Statistic:',T55,F7.3,
     1/' ',T10,'Reduced Ballistic Equation Multiplier =',T55,F7.3)
 1115 FORMAT(1H ,/'     Regression model form:',
     1      /'                  c1  c2   c3     c4    c5',
     1      /'      P* = P+1 =  e  *V  *Tb  *cosO  *DIA',
     1     /,'      Reduced Ballistic Limit Equation:',
     1     /,'                 z2   z3     z4',
     1     /,'      DIA = z1*V  *Tb  *cosO',
     1    //,'      The regression model coefficients are:')
 1125 FORMAT(1H ,T30,'Z(',I1,')=',E14.5)
 1130 FORMAT(1H ,/'     P* Critical =',F10.4,
     1 /'     The Reduced Ballistic Limit Equation coefficients are:')
      END
C
C        THIS IS A SUB-PROGRAM ,CALLED STAT.FOR, THAT DETERMINES
C        STATISTICAL PARAMETERS OF A REGRESSION OF DATA WITH
C        RESPECT TO ACTUAL DATA, AND PRODUCES RESIDUAL DATA FILES

C        FOR EXAMINATION, IF REQUESTED.
C      WRITTEN BY WILLIAM H. JOLLY   APRIL 2, 1992
C
      SUBROUTINE STAT(A,MX,N,NR,AV,PRT,SE2,R2,FC,SYY,YBAR,SSTC,SSREG)
      DIMENSION A(NR,*), AV(*)
      DOUBLE PRECISION S
      INTEGER PRT
C        INITIALIZE
      FC = 0.0
      NM1 = N-1
      NP1 = N+1
```

```
C       AV IS THE SET OF COEFFICIENTS DETERMINED FROM A
C       LINEAR REGRESSION.
C       STATISTICAL CALCULATIONS
C                   STATISTICAL QUANTITIES
C            YBN   = N*YBAR**2
C            SSTC = SS (TOTAL CORRECTED)
C            SSREG= SS (REGRESSION) OR b'X'y - YBN
C            RS    = RESIDUAL
C            SE2   = MEAN SQUARE OF THE RESIDUAL
C            FC    = F STATISTIC (MS_REGRESSION/MS_RESIDUAL)
C            R2    = CORRELATION COEFFICIENT
      BXY = 0.0
      DO 65 J=1,N
      S=0.0D0
      DO 66 K=1,MX
   66     S = S + A(K,J)*A(K,NP1)
      BXY = BXY + S*AV(J)
   65 CONTINUE
      YBN=FLOAT(MX)*YBAR**2
      SSTC=SYY-YBN
      SSREG = BXY - YBN
      SE2 = (SYY-BXY)/FLOAT(MX-N)
      FC = SSREG/NM1/SE2
      R2 = SSREG/SSTC
      IF(PRT.EQ.1) GOTO 999
      WRITE(7,1100)
C      CALCULATE PREDICTIONS, RESIDUALS, CORRELATION, AND F STATISTIC
      DO 90 I=1,MX
      YHAT=0.0
      DO 95 J=1,N
   95     YHAT = YHAT + A(I,J)*AV(J)
        RES = A(I,NP1) - YHAT
      WRITE(7,1000) A(I,NP1),YHAT,RES,(A(I,K),K=2,N)
   90 CONTINUE
 1000 FORMAT(1H ,7E16.6)
 1100 FORMAT(1H ,T3,'ACTUAL',T14,'PREDICTED',T25,'RESIDUAL',
     1            T36,'INDEPENDENT VARIABLES')
  999 RETURN
      END
C
      SUBROUTINE CHOL(G,M,ERR)
C         THIS ROUTINE FINDS THE CHOLESKI DECOMPOSITION MATRIX
C         OF A SYMMETRIC POSITIVE DEFINITE MATRIX.
C           DIAGONOSTIC (ERR) IS SET TO ONE IF MATRIX IS NOT
C           POSITIVE DEFINITE.
C         THE CHOLESKI FACTOR IS FOUND IN THE LOWER TRIANGULAR SECTION
C         OF THE INPUT MATRIX G.  THE UPPER TRIANGULAR SECTION IS NOT
C         CHANGED.
      DOUBLE PRECISION S
      DIMENSION G(M,*)
      INTEGER ERR, N
      N=M-1
      ERR = 0
C     FIRST COLUMN DECOMPOSITION (to avoid un-necessary if statement)
      G(1,1) = SQRT(G(1,1))
      DO 100 I = 2,N
  100   G(I,1) = G(I,1)/G(1,1)
C     COMPLETION OF THE DECOMPOSITION
      DO 200 J = 2,N
      S = 0.0D0
      DO 210 I = 1,J-1
  210     S = S + G(J,I)**2
      IF(G(J,J).GT.S) GOTO 215
        ERR=1
```

```
        GOTO 200
 215    G(J,J) = SQRT(G(J,J)-S)
      DO 230 I = J+1,N
        S = 0.0D0
        DO 220 K = 1,J-1
 220        S = S + G(I,K)*G(J,K)
 230    G(I,J) = (G(I,J)-S)/G(J,J)
 200  CONTINUE
 999  RETURN
      END
C
      SUBROUTINE NORM(A,NDP,N,NR,SYY,YBAR,C,B)
C        THIS ROUTINE WILL COMPUTE THE N BY N+1
C        NORMAL MATRIX C=X'X AND B=X'b;
C        AND DETERMINE SOME STATISITCAL PARAMETERS.
      DIMENSION A(NR,*), C(N,*), B(*)
      DOUBLE PRECISION S
      DO 10  I=1,N
      DO 10  J=I,N
      S = 0.0D0
      DO 15  K=1,NDP
   15     S = S + A(K,I)*A(K,J)
      C(I,J) = S
      IF(J.LT.N)  C(J,I)=C(I,J)
   10 CONTINUE
      DO 20 I=1,N-1
   20   B(I)=C(I,N)
      SYY=C(N,N)
      YBAR=B(1)/C(1,1)
      RETURN
      END
C
      SUBROUTINE FSOLR(C,D,N,M)
      DOUBLE PRECISION S
      DIMENSION C(M,*), D(*)
      D(1)=D(1)/C(1,1)
      DO 100 I=2,N
      S = 0.0D0
      DO 110 J=1,I-1
 110      S = S + C(I,J)*D(J)
 100  D(I) = (D(I)-S)/C(I,I)
      RETURN
      END
C
      SUBROUTINE BSOLRT(A,B,N,M)
      DOUBLE PRECISION S
      DIMENSION A(M,*), B(*)
      B(N)=B(N)/A(N,N)
      DO 100 I=N-1,1,-1
      S = 0.0D0
      DO 110 J=I+1,N
 110      S = S + A(J,I)*B(J)
 100  B(I) = (B(I)-S)/A(I,I)
      RETURN
      END
C
      SUBROUTINE CI(C,X,N,S2,DELTA)
C      THIS SUBROUTINE DETERMINES THE CONFIDENCE INTERVAL FOR A
C      GIVEN X BASED UPON THE NORMAL MATRIX C AND THE ESTIMATED
C      VARIATION S2.
      DOUBLE PRECISION S, S1
      DIMENSION C(N,*),X(*)
      S1=0.0D0
      DO 20  I=1,N
```

```
      S = 0.0D0
      DO 10  J=1,N
   10   S = S + C(I,J)*X(J)
   20 S1 = S1 + S*X(I)
      DELTA=SQRT(S2*DABS(S1))
      RETURN
      END
C
      SUBROUTINE CINV(A,X,N,NR,B)
C      THIS SUBROUTINE CALCULATES THE INVERSE OF A MATRIX, GIVEN
C      ITS CHOLESKI DECOMPOSITION (LOWER TRIANGULAR).
      DIMENSION A(NR,*), X(N,*), B(*)
      DO 10 I=1,N
        DO 20 J=1,N
          B(J)=0.0
   20   IF(I.EQ.J) B(J)=1.0
      CALL FSOLR(A,B,N,NR)
      CALL BSOLRT(A,B,N,NR)
      DO 30 J=1,N
   30   X(I,J)=B(J)
   10 CONTINUE
      RETURN
      END
```

APPENDIX B.2 - MULT.FOR, MULTIPLE STEPWISE REGRESSION CODE

```
C
C      THIS IS A MULTIPLE REGRESSION PROGRAM CALLED MULT.FOR
C
C      WRITTEN BY WILLIAM H. JOLLY   NOV 7, 1991
C
      PARAMETER (M=500,N=12)
C      M IS THE MAXIMUM SIZE OF THE ARRAYS
C               CORRESPONDING TO THE NUMBER OF DATA POINTS.
C      N IS THE MAXIMUM SIZE OF THE ARRAYS
C               CORRESPONDING TO THE NUMBER OF DEPENDENT
      REAL X(M,N),SV(99,17),RD(M,N),VAL(50,2),C(N)
      INTEGER NI(2), Q
      CHARACTER *20 INFILE,DATFILE,OUTFILE,ANS,NO,YES
      DATA NO/"n"/,YES/"y"/,NI/1,1/
  1   FORMAT(A20)
      PRINT*,'        STEPWISE NON-LINEAR DATA REGRESSION PROGRAM'
      PRINT*,' '
      PRINT*,'                            c2   c3     c4    c5'
      PRINT*,'               P+1 =  c1*V   *Tb  *cosO  *DIA'
      PRINT*,' '
      PRINT*,'THE RESULTING EQUATION IS GIVEN AS:'
      PRINT*,' '
      PRINT*,'                           z2   z3     z4'
      PRINT*,'               DIA = z1*V  *Tb  *cosO'
      PRINT*,' '
      PRINT*,'REQUIRED INPUT DATASET FORMAT:'
      PRINT*,'* RWTHK'
      PRINT*,'  WITPEN  RWPEND  VEL  BUMGAGE  OBLIQUE  PRJDIA'
      PRINT*,'     WHERE:'
      PRINT*,'           WITPEN = NO. OF WITNESS PLATES PENETRATED'
      PRINT*,'           RWPEND = DEPTH OF REAR WALL PENETRATION (cm)'
      PRINT*,'           VEL    = VELOCITY OF PROJECTILE (km/s)'
      PRINT*,'           BUMGAGE= THICKNESS OF BUMPER (cm)'
      PRINT*,'           OBLIQUE= ANGULAR DEVIATION FROM BUMPER SURFACE'
      PRINT*,'                    NORMAL VECTOR TO IMPACT TRAJECTORY'
      PRINT*,'                    (degrees)'
      PRINT*,'           PRJDIA = PROJECTILE DIAMETER (cm)'
      PRINT*,' '
 30   PRINT*,'DO YOU WANT AN OUTPUT FILE (y/n)?'
      PRINT*,'>'
      READ(*,1) ANS
      IF(ANS.EQ.NO) GOTO 40
      IF(ANS.NE.YES) GOTO 30
 40   PRINT*,' '
      PRINT*,'ENTER THE INPUT DATA FILENAME, NO EXTENSION:'
      PRINT*,'   THE EXTENSION OF .dat IS ASSUMMED.'
      PRINT*,'>'
      READ(*,1) INFILE
      L=LEN_TRIM(INFILE)
      DATFILE = INFILE(:L)//'.dat'
      PRINT*,' '
      PRINT*,'INPUT FILE IS:',DATFILE
      IF(ANS.EQ.NO) GOTO 42
      OUTFILE = INFILE(:L)//'.out'
      PRINT*,' '
      PRINT*,'THE OUTPUT FILE IS:',OUTFILE
      OPEN(UNIT=6,FILE=OUTFILE)
 42   OPEN(UNIT=5,FILE=DATFILE)

C   *******************************************************************
C      THIS IS THE ALTERABLE CODE SECTION FOR THE REGRESSION
C      ROUTINE.  READ THE DATA, OPERATE ON IT, AND LOAD IT INTO
C      THE X MATRIX.  THEN CALL RSTAT.
C   *******************************************************************
```

```
   45 READ(5,*) NN, RWT
      RWTHK=RWT*.9999
      NC = 6
      NX = NC - 1
      NT = NC - 2
      NT1= NC - 3
      NT2= NC - 4
      NC1= NC + 1
      NC2= NC + 2
C       NC IS NUMBER OF COLUMNS IN THE INPUT DATASET
C       NX IS THE NUMBER OF TERMS ON RHS.  INCLUCES A0.
C       NT IS THE NUMBER OF INDEPENDENT VARIABLES. X1, X2, X3, ...
C       NP IS THE NUMBER OF DATA POINTS IN THE INPUT FILE.
C       RWTHK IS THE REAR WALL THICKNESS.  THE VALUE IS REDUCED TO ASSURE
C       INTEGER ROUNDUP TO ONE IN THE PENETRATION PARAMETER EQUATION.
C       SV(I,J) IS THE SOLUTION ARRAY WHERE I REPRESENTS REGRESSIONS
C       AND J ARE THE ATTRIBUTES OF EACH REGRESSION AS FOLLOWS:
C
C J=    1-5  C(I)
C       1,6     CRITICAL PENETRATION VALUE
C       N,6 AND N,7   CONSTANT VALUE BACK CONVERTED
C       8-11 or (NC2-NC2+3) IS Z(I)
C       12   SE2 - SUM OF SQUARES OF RESIDUALS
C       14   FC - F DISTRIBUTION STATISTIC
C       13   R2 - MULTIPLE CORRELATION COEFFICIENT
C       16   N  - NUMBER OF POINTS REGRESSED
C       15   ERR - ERROR OCCURED ON FINDING INVERSE IF NOT ZERO
C       17   RES - TOTAL SUM OF THE RESIDUALS (MUST BE NEAR ZERO)
C
      I=1
   50 READ(5,*,END=55)(RD(I,J),J=1,6)
      I=I+1
      GOTO 50

   55 NP=I-1
      IF(NP.GT.NC) GOTO 60
      PRINT*,' '
      PRINT*,'NOT ENOUGH DATA POINTS TO PERFORM A REGRESSION.'
      PRINT*,'TERMINATING PROGRAM!'
      STOP
   60 DO 75 I=1,NP
      RD(I,NC1)=RD(I,2)*2.851+(RD(I,1)+0.5*INT(RD(I,2)/RWTHK))
     1                *0.14202+1.0
      X(I,1)=1.0
      DO 70 J=2,NC
   70   X(I,J)=ALOG(RD(I,J+1))
   75 CONTINUE

      CALL RSTAT(X,NP,NX,M,C,SV(1,12),SV(1,13),
     1           SV(1,14),DTRM,SV(1,15),SV(1,17))

C       LOAD THE SOLUTION ARRAY SV(I,J)

      DO 76 I=1,NX
   76   SV(1,I)=C(I)
      SV(1,NC)=(RWT*2.851+1.0)
      SV(1,16)=FLOAT(NP)

C       CALCULATE EXPONENTS IN NONLINEAR EQN SOLVED FOR DIA

      SV(1,NC2)=(SV(1,NC)/EXP(SV(1,1)))**(1/SV(1,NX))
      DO 77 J=NC2+1,NC2+3
   77   SV(1,J)=-SV(1,J-NC1)/SV(1,NX)
```

```
C       CATAGORIZE THE X ARRAY
C         FIND THE NUMBER AND VALUE OF THE UNIQUE ELEMENTS
C         OF BUMGAGE AND OBLIQUITY.
C          NI(1) = NUMBER OF T'S, VAL(1-NI(1),1)=VALUES OF T'S
C          NI(2) = NUMBER OF O'S, VAL(1-NI(2),2)=VALUES OF O'S

      VAL(1,1)=X(1,NT1)
      VAL(1,2)=X(1,NT)
      DO 125 I=2,NP
      DO 110 Q=1,2
        NXP=Q+NT1-1
        K=1
 90       IF(X(I,NXP).EQ.VAL(K,Q)) GOTO 110
        K=K+1
        IF(K.LE.NI(Q)) GOTO 90
        NI(Q)=NI(Q)+1
        VAL(NI(Q),Q)=X(I,NXP)
 110    CONTINUE
 125  CONTINUE

C       SORT VAL(I,Q) INTO DECENDING ORDER

      DO 126 Q=1,2
      DO 126 I=1,NI(Q)-1
      DO 126 J=I+1,NI(Q)
        IF(VAL(I,Q).GT.VAL(J,Q)) GOTO 126
        Z=VAL(I,Q)
        VAL(I,Q)=VAL(J,Q)
        VAL(J,Q)=Z
 126  CONTINUE

C       PERFORM NI(1)+NI(2) REGRESSIONS FOR CONSTANT OBLIQUITY
C       AND CONSTANT THICKNESS, RESPECTIVELY.

      NUMR=NI(1)+NI(2)+1
      PRINT*,' '
      PRINT*,'  THE NUMBER OF UNIQUE THICKNESSES:',NI(1)
      PRINT*,'  THE NUMBER OF UNIQUE OBLIQUITIES:',NI(2)

      JJ=1
      DO 160 Q=1,2
      NXP = Q+2
      DO 160 L=1,NI(Q)
        NPTS=0
        DO 140 I=1,NP
        IF(X(I,NXP).NE.VAL(L,Q)) GOTO 140
          NPTS=NPTS+1
          DO 145 J=1,NC
            IF(J.LT.NXP) RD(NPTS,J)=X(I,J)
 145          IF(J.GT.NXP) RD(NPTS,J-1)=X(I,J)
 140      CONTINUE

        IF(NPTS.EQ.0) GOTO 160
        JJ=JJ+1
        SV(JJ,16)=FLOAT(NPTS)
        IF(Q.EQ.1) TEMP=EXP(VAL(L,1))
        IF(Q.EQ.2) TEMP=ACOS(EXP(VAL(L,2)))*180.0/3.1415927
        SV(JJ,NC)=TEMP
        IF(NPTS.LE.NX) GOTO 160
 146      CALL RSTAT(RD,NPTS,NT,M,C,SV(JJ,12),SV(JJ,13),
     1                SV(JJ,14),DTRM,SV(JJ,15),SV(JJ,17))
        IF(SV(JJ,15).NE.0) GOTO 160
        DO 150 I=1,NT
 150        SV(JJ,I)=C(I)
```

```
      SV(JJ,NC2)=(SV(1,NC)/EXP(SV(JJ,1)))**(1/SV(JJ,NT))
      DO 155 K=NC2+1,NC2+2
155       SV(JJ,K)=-SV(JJ,K-NC1)/SV(JJ,NT)
160   CONTINUE

C       PERFORM PV REGRESSIONS

      JJ=NUMR
      DO 250 L=1,NI(2)
      DO 250 Q=1,NI(1)
      NPTS=0
      DO 200 I=1,NP
        IF((X(I,NT1).EQ.VAL(Q,1)).AND.(X(I,NT).EQ.VAL(L,2))) THEN
          NPTS=NPTS+1
          DO 210 J=1,NC
            IF(J.LT.NT1) RD(NPTS,J)=X(I,J)
210           IF(J.GT.NT) RD(NPTS,J-2)=X(I,J)
        ENDIF
200       CONTINUE

        IF(NPTS.EQ.0) GOTO 250
        JJ=JJ+1
        SV(JJ,16)=FLOAT(NPTS)
        SV(JJ,NC)=ACOS(EXP(VAL(L,2)))*180.0/3.1415927
        SV(JJ,NC1)=EXP(VAL(Q,1))
        IF(NPTS.LE.NT) GOTO 250
220       CALL RSTAT(RD,NPTS,NT1,M,C,SV(JJ,12),SV(JJ,13),
     1              SV(JJ,14),DTRM,SV(JJ,15),SV(JJ,17))
        IF(SV(JJ,15).NE.0) GOTO 250
        DO 230 I=1,NT1
230       SV(JJ,I)=C(I)
        SV(JJ,NC2)=(SV(1,NC)/EXP(SV(JJ,1)))**(1/SV(JJ,NT1))
        SV(JJ,NC2+1)=-SV(JJ,NT2)/SV(JJ,NT1)
250   CONTINUE
      NPV = JJ
      PRINT*,' '
      PRINT*,'  TOTAL NUMBER OF REGRESSIONS PERFORMED:',JJ

C  ******************************************************************
C       WELCOME TO THE OUTPUT SECTION
C  ******************************************************************

C HEADER
      WRITE(6,995) DATFILE,NX,NPV
C FIRST REGRESSION
      WRITE(6,1105)(SV(1,J),J=1,NX)
      WRITE(6,1110)
      WRITE(6,1105)(SV(1,J),J=NC2,NC2+3)
      WRITE(6,1115)
      WRITE(6,1109)(SV(1,J),J=12,14),INT(SV(1,15)),
     1             INT(SV(1,16)),SV(1,17)
C CONST THICKNESS
      WRITE(6,900) NI(1)
      DO 905 I=2,NI(1)+1
905   WRITE(6,1106)I,SV(I,NC),(SV(I,J),J=1,NT)
      WRITE(6,1111)
      DO 910 I=2,NI(1)+1
910   WRITE(6,1106)I,SV(I,NC),(SV(I,J),J=NC2,NC2+2)
      WRITE(6,1116)
      DO 915 I=2,NI(1)+1
915   WRITE(6,1108)I,(SV(I,J),J=12,14),INT(SV(I,15)),
     1             INT(SV(I,16)),SV(I,17)
C CONST OBLIQUITY
      WRITE(6,1000) NI(2)
```

```
      DO 1005 I=NI(1)+2,NUMR
 1005 WRITE(6,1107)I,SV(I,NC),(SV(I,J),J=1,NT)
      WRITE(6,1112)
      DO 1010 I=NI(1)+2,NUMR
 1010 WRITE(6,1107)I,SV(I,NC),(SV(I,J),J=NC2,NC2+2)
      WRITE(6,1116)
      DO 1015 I=NI(1)+2,NUMR
 1015 WRITE(6,1108)I,(SV(I,J),J=12,14),INT(SV(I,15)),
     1               INT(SV(I,16)),SV(I,17)
C PV REGRESSION OUTPUT
      WRITE(6,1001) NPV
      DO 1020 I=NUMR+1,NPV
 1020 WRITE(6,1107)I,SV(I,NC),SV(I,NC1),(SV(I,J),J=1,NT1)
      WRITE(6,1002)
      DO 1025 I=NUMR+1,NPV
 1025 WRITE(6,1107)I,SV(I,NC),SV(I,NC1),(SV(I,J),J=NC2,NC2+1)
      WRITE(6,1116)
      DO 1030 I=NUMR+1,NPV
 1030 WRITE(6,1108)I,(SV(I,J),J=12,14),INT(SV(I,15)),
     1               INT(SV(I,16)),SV(I,17)

  995 FORMAT(1H ,//' ',T10,'PROGRAM: mult.for',T45,'INPUT FILENAME: ',
     1        A20,/' ',T10,'STEPWISE NON-LINEAR REGRESSION ANALYSIS',
     1        //'                                c2    c3      c4     c5',
     1         /'                          P+1 =  c1*V   *Tb   *cosO   *DIA',
     1            //' ',T9,I2,' VARIABLES',
     1              ' STEPWISE REGRESSED OVER',I5,'    MODELS',
     1             //' ',T10,'=> REGRESSION OVER ALL VARIABLES.',
     1              /' ',T13,'THE LINEAR COEFFICIENTS ARE:',
     1  /' ',T14,'C(1)',8X,'C(2)',8X,'C(3)',8X,'C(4)',8X,'C(5)')
  900 FORMAT(1H ,/' ',T10,'=> CONSTANT THICKNESS REGRESSIONS OVER',I2,
     1     ' VARIABLES.',/' ',T13,'THE LINEAR COEFFICIENTS ARE:',
     1  /' ',T14,'R NO.',5X,'THK',8X,'C(1)',8X,'C(2)',8X,'C(3)',
     1       8X,'C(4)')
 1000 FORMAT(1H ,/' ',T10,'=> CONSTANT OBLIQUITY REGRESSIONS OVER',I2,
     1     ' VARIABLES.',/' ',T14,'THE LINEAR COEFFICIENTS ARE:',
     1  /' ',T14,'R NO.',4X,'cosO',8X,'C(1)',8X,'C(2)',8X,'C(3)',
     1       8X,'C(4)')
 1001 FORMAT(1H ,/' ',T10,'=> CONSTANT OBLIQUITY AND THICKNESS',
     1     ' REGRESSIONS OVER',I2,
     1     ' VARIABLES.',/' ',T14,'THE LINEAR COEFFICIENTS ARE:',
     1  /' ',T14,'R NO.',4X,'cosO',9X,'THK',8X,'C(1)',8X,'C(2)',
     1       8X,'C(3)')
 1002 FORMAT(1H ,T13,'THE NON-LINEAR COEFFICIENTS ARE:',
     1  /' ',T14,'R NO.',4X,'cosO',9X,'THK',8X,'Z(1)',8X,'Z(2)')
 1100 FORMAT(1H ,'**MATRIX SOLUTION ERROR, CONVERGENCE NOT COMPLETE OR',
     1           /'  MATRIX IS SINGULAR.   ERROR INDICATOR = ',I3)
 1105 FORMAT(1H ,T8,6F12.4)
 1106 FORMAT(1H ,T11,I5,5F12.4)
 1107 FORMAT(1H ,T11,I5,F12.0,4F12.4)
 1108 FORMAT(1H ,T11,I5,3F12.4,I6,I7,E13.3)
 1109 FORMAT(1H ,T8,3F12.4,I7,I8,E11.3)
 1110 FORMAT(1H ,T13,'THE NON-LINEAR COEFFICIENTS ARE:',
     1               /' ',T14,'Z(1)',8X,'Z(2)',8X,'Z(3)',8X,'Z(4)')
 1111 FORMAT(1H ,T13,'THE NON-LINEAR COEFFICIENTS ARE:',
     1          /' ',T14,'R NO.',5X,'THK',8X,'Z(1)',8X,'Z(2)',8X,'Z(3)')
 1112 FORMAT(1H ,T13,'THE NON-LINEAR COEFFICIENTS ARE:',
     1          /' ',T14,'R NO.',4X,'cosO',8X,'Z(1)',8X,'Z(2)',8X,'Z(3)')
 1115 FORMAT(1H ,T13,'STATISTICAL RESULTS:',
     1   /' ',T14,'MS_RES',7X,'CORR',4X,'F STATISTIC',3X,'ERR',4X,'NUM',
     1   4X,'SUM R')
 1116 FORMAT(1H ,T13,'STATISTICAL RESULTS:',
     1   /' ',T14,'R NO.',3X,'MS_RES',7X,'CORR',4X,'F STATISTIC',
     1   2X,'ERR',3X,'NUM',6X,'SUM R')
```

```
      STOP
      END

C         THIS IS A SUB-PROGRAM CALLED RSTAT.FOR
C         WHERE A DATA MATRIX IS SOLVED FOR A LEAST SQUARES FIT
C         AND STATISTICAL INFERENCES ARE MADE ON THE 'GOODNESS OF
C         FIT' OF THAT DATA TO THE GIVEN MODEL.
C
C       WRITTEN BY WILLIAM H. JOLLY   OCT 23, 1991
C
      SUBROUTINE RSTAT(A,MX,N,NR,AV,SE2,R2,FC,DET,ERR,RES)
      DOUBLE PRECISION ADP,S,SD,AH,Z,P,DMIN
      DIMENSION A(NR,*),AV(*),ADP(12,12)

C         INITIALIZE
      SD = 1.0D0
      ERR = 0.0
      JMAX = N+1
      NM1 = N-1
      DMIN = .5D-7
      FC=0.0
      YBAR=0.0
      RES=0.0

C         CALCULATE NORMAL MATRIX B

      DO 10  I=1,JMAX
      DO 10  J=I,JMAX
      S = 0.0D0
      DO 15  K=1,MX
      IF((I.EQ.1).AND.(J.EQ.1)) YBAR = YBAR + A(K,JMAX)
   15 S = S + A(K,I)*A(K,J)
      ADP(I,J) = S
      IF(J .LT. JMAX)  ADP(J,I)=ADP(I,J)
   10 CONTINUE

      YBAR=YBAR/FLOAT(MX)
      SYY=ADP(JMAX,JMAX)

C         SOLVE SYSTEM OF EQUATIONS
C       ERR IS THE ERROR INDICATOR (IF DET IS ZERO, ERR IS NOT ZERO)
C       DET IS THE VALUE OF THE DETERMINANT
C       AV IS THE SOLUTION VECTOR

C       NORMAL MATRIX SOLUTION BY GAUSSIAN ELIMINATION

      DO 20  K=1,NM1
      KP1 = K+1
C       REORDER EQUATIONS
      IH = K
      AH = DABS(ADP(K,K))
      DO 25  I=K,N
      IF(DABS(ADP(I,K)) .GT. AH) THEN
      IH = I
      AH = DABS(ADP(I,K))
      ENDIF
   25 CONTINUE
C       CHANGE ORDER
      IF(IH .NE. K) THEN
      SD = -SD
      DO 30  J=1,JMAX
      Z = ADP(K,J)
      ADP(K,J) = ADP(IH,J)
```

```
   30   ADP(IH,J) = Z
      ENDIF
C       ZERO BELOW DIAGONAL
      DO 35  J=KP1,JMAX
      IF(DABS(ADP(K,K)) .GT. DMIN)  GOTO 35
      ERR = FLOAT(K-1)
      GOTO 999
   35 ADP(K,J) = ADP(K,J)/ADP(K,K)
      DO 20  I=KP1,N
      Z = ADP(I,K)
      DO 20  J=KP1,JMAX
   20 ADP(I,J) = ADP(I,J) - Z*ADP(K,J)
C       DETERMINANT
   40 P = 1.0D0
      DO 45  K=1,N
   45 P = P*ADP(K,K)
      DET = SD*P
C          CALCULATE UNKNOWNS BY BACK SUBSTITUTION
   50 ADP(N,1) = ADP(N,JMAX)/ADP(N,N)
      AV(N) = ADP(N,1)
      DO 55  K=1,NM1
      I = N-K
      IP1 = I+1
      S = 0.0D0
      DO 60  J=IP1,N
   60 S = S + ADP(I,J)*ADP(J,1)
      ADP(I,1) = ADP(I,JMAX) - S
   55 AV(I) = ADP(I,1)

C       STATISTICAL CALCULATIONS
C                       STATISTICAL QUANTITIES
C            ADP(1,2) = b' X' y
C            YBN      = N*YBAR**2
C            SSTC     = SS (TOTAL CORRECTED)
C            SSREG    = SS (REGRESSION) OR b'X'y - YBN
C            RS       = RESIDUAL = SSTC-SSREG = sum(y(i)-yhat)^2
C            SE2      = MEAN SQUARE OF THE RESIDUAL
C            FC       = F STATISTIC (MS_REGRESSION/MS_RESIDUAL)
C            R2       = CORRELATION COEFFICIENT

      ADP(1,2) = 0.0D0
      DO 65 J=1,N
      S=0.0D0
      DO 66 K=1,MX
   66     S = S + A(K,J)*A(K,JMAX)
      ADP(1,2) = ADP(1,2) + S*ADP(J,1)
   65 CONTINUE

      DO 90 I=1,MX
      YHAT=0.0
      DO 95 J=1,N
   95     YHAT = YHAT + A(I,J)*AV(J)
        RES = RES + (A(I,JMAX)-YHAT)
   90 CONTINUE

      YBN=FLOAT(MX)*YBAR**2
      SSREG = ADP(1,2) - YBN
      SSTC = SYY - YBN
      SE2= (SYY-ADP(1,2))/FLOAT(MX-N)
      FC = SSREG/FLOAT(NM1-1)/SE2
      R2 = SSREG/SSTC

  999 RETURN
      END
```

APPENDIX C

RESULTS OF REGRESSIONS

APPENDIX C.1 - FINAL ANALYSIS OF 221 SHOTS

PROGRAM: mult.for INPUT FILENAME: smli.dat

STEPWISE NON-LINEAR REGRESSION ANALYSIS

$$P+1 = c1*V^{c2} *Tb^{c3} *cosO^{c4} *DIA^{c5}$$

5 VARIABLES STEPWISE REGRESSED OVER 22 MODELS

=> REGRESSION OVER ALL VARIABLES.

THE LINEAR COEFFICIENTS ARE:

C(1)	C(2)	C(3)	C(4)	C(5)
.8532	-.0547	-.0815	.2238	.5268

THE NON-LINEAR COEFFICIENTS ARE:

Z(1)	Z(2)	Z(3)	Z(4)
.6729	.1038	.1546	-.4249

STATISTICAL RESULTS:

MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
.0306	.2993	30.7508	0	221	.124E-05

=> CONSTANT THICKNESS REGRESSIONS OVER 5 VARIABLES.

THE LINEAR COEFFICIENTS ARE:

R NO.	THK	C(1)	C(2)	C(3)	C(4)
2	.2032	.9852	-.0940	-.2034	.6236
3	.1600	1.1934	-.1523	.1227	.6904
4	.1270	.0000	.0000	.0000	.0000
5	.1016	1.5330	-.2429	.5228	.7023
6	.0813	1.3581	-.0945	.5503	.5894

THE NON-LINEAR COEFFICIENTS ARE:

R NO.	THK	Z(1)	Z(2)	Z(3)
2	.2032	.5792	.1507	.3262
3	.1600	.4516	.2207	-.1777
4	.1270	.0000	.0000	.0000
5	.1016	.2822	.3459	-.7444
6	.0813	.2980	.1604	-.9337

STATISTICAL RESULTS:

R NO.	MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
2	.0278	.3565	13.2968	0	52	.976E-06
3	.0258	.3760	24.1072	0	84	.472E-05
4	.0000	.0000	.0000	2	17	.000E+00
5	.0325	.4350	16.5511	0	47	-.214E-05
6	.0213	.6088	13.2286	0	21	-.603E-06

=> CONSTANT OBLIQUITY REGRESSIONS OVER 3 VARIABLES.

THE LINEAR COEFFICIENTS ARE:

R NO.	cosO	C(1)	C(2)	C(3)	C(4)
7	0.	.6160	-.1699	-.2977	.5694
8	45.	.7627	-.0333	-.1605	.7783
9	65.	1.3686	-.1137	.2218	.5726

THE NON-LINEAR COEFFICIENTS ARE:

R NO.	cosO	Z(1)	Z(2)	Z(3)
7	0.	1.0514	.2983	.5228
8	45.	.8591	.0428	.2063
9	65.	.2824	.1986	-.3874

STATISTICAL RESULTS:

R NO.	MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
7	.0364	.1748	9.0018	0	89	-.151E-05
8	.0141	.6437	83.1045	0	96	-.117E-05
9	.0248	.3760	9.6427	0	36	.582E-07

=> CONSTANT OBLIQUITY AND THICKNESS REGRESSIONS OVER22 VARIABLES.

THE LINEAR COEFFICIENTS ARE:

R NO.	cosO	THK	C(1)	C(2)	C(3)
10	0.	.2032	.9516	-.2065	-.0368
11	0.	.1600	1.1973	-.1674	.6571

12	0.	.1016	1.7569	-.5043	.3480
13	0.	.0813	1.5771	-.2595	.5344
14	45.	.2032	1.0073	-.0237	.7578
15	45.	.1600	1.0358	-.0120	.8604
16	45.	.1270	1.1005	.0156	.9833
17	45.	.1016	1.3983	-.1725	.8616
18	45.	.0813	2.1688	-.6588	.3318
19	65.	.2032	.0000	.0000	.0000
20	65.	.1600	.8301	-.0285	.7158
21	65.	.1016	.9570	-.1350	1.0129
22	65.	.0813	.8361	.0118	.8876

THE NON-LINEAR COEFFICIENTS ARE:

R NO.	cosO	THK	Z(1)	Z(2)
10	0.	.2032	4230.6640	-5.6159
11	0.	.1600	.4312	.2547
12	0.	.1016	.0409	1.4490
13	0.	.0813	.1746	.4856
14	45.	.2032	.6197	.0313
15	45.	.1600	.6346	.0139
16	45.	.1270	.6290	-.0159
17	45.	.1016	.4169	.2002
18	45.	.0813	.0101	1.9858
19	65.	.2032	.0000	.0000
20	65.	.1600	.7717	.0399
21	65.	.1016	.7346	.1333
22	65.	.0813	.8059	-.0133

STATISTICAL RESULTS:

R NO.	MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
10	.0529	.0380	.5134	0	16	-.949E-07
11	.0312	.2634	18.9564	0	56	.114E-05
12	.0622	.1476	1.5587	0	12	.124E-08
13	.0006	.9335	28.0729	0	5	.566E-07
14	.0165	.6292	50.9087	0	33	.628E-06
15	.0102	.7748	58.4749	0	20	-.447E-06
16	.0129	.6440	25.3214	0	17	-.535E-06
17	.0149	.7221	41.5778	0	19	.203E-06
18	.0111	.4341	3.0680	0	7	-.152E-06
19	.0000	.0000	.0000	0	3	.000E+00
20	.0225	.3860	3.1440	0	8	.133E-06
21	.0172	.4891	12.4470	0	16	-.287E-06
22	.0369	.5291	6.7409	0	9	-.910E-07

APPENDIX C.2 - SINGLE REGRESSION OUTPUT

MODEL: mdreg.for   INPUT FILENAME: smli.dat

NON-LINEAR REGRESSION ANALYSIS for 221 observations
using 4 variables resulting in 216 Degrees of Freedom:

Regression model form:

$$P^* = P+1 = e^{c1} *V^{c2} *Tb^{c3} *\cos O^{c4} *DIA^{c5}$$

Reduced Ballistic Limit Equation:

$$DIA = z1*V^{z2} *Tb^{z3} *\cos O^{z4}$$

The regression model coefficients are:

C(1)=	.85325E+00
C(2)=	-.54692E-01
C(3)=	-.81457E-01
C(4)=	.22383E+00
C(5)=	.52678E+00

P* Critical = 1.9052

The Reduced Ballistic Limit Equation coefficients are:

Z(1)=	.67293E+00
Z(2)=	.10382E+00
Z(3)=	.15463E+00
Z(4)=	-.42491E+00

ANOVA

Source	df	SS	MS	F Value
Regression	4	.28211E+01	.70527E+00	.23064E+02
Residual	216	.66051E+01	.30579E-01	
Total Corrected	220	.94262E+01		

Multiple Correlation Coefficient:	(R2)=	.29928E+00
Estimate of Variance:	(SE2)=	.30579E-01

Mean Vector (Xo):

Velocity	Tb	cosO	Dia
.52852E+01	.14156E+00	.74763E+00	.66457E+00

Confidence Interval:

Predicted Mean Solution: P*(Xo)=	1.899
95% Confidence Interval on P* is: [1.855,	1.943]
The specified value of the t Statistic:	1.960
Reduced Ballistic Equation Multiplier =	1.045

```
MODEL: mdreg.for     INPUT FILENAME: csmli0.dat

NON-LINEAR REGRESSION ANALYSIS for   89 observations
using    3   variables resulting in   85 Degrees of Freedom:

Regression model form:
             c1  c2   c3     c4    c5
P* = P+1 =  e  *V  *Tb  *cosO  *DIA
Reduced Ballistic Limit Equation:
          z2   z3     z4
DIA = z1*V  *Tb  *cosO

The regression model coefficients are:
                                C(1)=     .61603E+00
                                C(2)=    -.16986E+00
                                C(3)=    -.29769E+00
                                C(4)=     .56940E+00

P* Critical =    1.9052
The Reduced Ballistic Limit Equation coefficients are:
                                Z(1)=     .10514E+01
                                Z(2)=     .29832E+00
                                Z(3)=     .52282E+00
```

ANOVA

Source	df	SS	MS	F Value
Regression	3	.65562E+00	.21854E+00	.60009E+01
Residual	85	.30955E+01	.36418E-01	
Total Corrected	88	.37511E+01		

```
Multiple Correlation Coefficient:  (R2)=                .17478E+00
Estimate of Variance:              (SE2)=               .36418E-01

Mean Vector (Xo):
        Velocity         Tb            cosO             Dia
        .53941E+01    .15125E+00    .67391E+00

Confidence Interval:
    Predicted Mean Solution:         P*(Xo)=           1.949
    95% Confidence Interval on P* is:   [  1.872,    2.029]
    The specified value of the t Statistic:            1.987
    Reduced Ballistic Equation Multiplier =            1.073
```

MODEL: mdreg.for   INPUT FILENAME: csmli45.dat

NON-LINEAR REGRESSION ANALYSIS for 96 observations
using 3 variables resulting in 92 Degrees of Freedom:

Regression model form:

```
              c1  c2   c3     c4    c5
P* = P+1 =  e  *V  *Tb  *cosO  *DIA
```

Reduced Ballistic Limit Equation:

```
           z2   z3     z4
DIA = z1*V  *Tb  *cosO
```

The regression model coefficients are:

```
C(1)=    .76274E+00
C(2)=   -.33348E-01
C(3)=   -.16054E+00
C(4)=    .77835E+00
```

P* Critical = 1.9052

The Reduced Ballistic Limit Equation coefficients are:

```
Z(1)=    .85915E+00
Z(2)=    .42844E-01
Z(3)=    .20626E+00
```

ANOVA

Source	df	SS	MS	F Value
Regression	3	.23477E+01	.78258E+00	.55402E+02
Residual	92	.12995E+01	.14125E-01	
Total Corrected	95	.36473E+01		

Multiple Correlation Coefficient: (R2)= .64370E+00
Estimate of Variance: (SE2)= .14125E-01

Mean Vector (Xo):

Velocity	Tb	cosO	Dia
.52337E+01	.14507E+00	.63695E+00	

Confidence Interval:
- Predicted Mean Solution: P*(Xo)= 1.947
- 95% Confidence Interval on P* is: [1.901, 1.995]
- The specified value of the t Statistic: 1.984
- Reduced Ballistic Equation Multiplier = 1.031

```
MODEL: mdreg.for     INPUT FILENAME: csmli65.dat

NON-LINEAR REGRESSION ANALYSIS for   36 observations
using    3   variables resulting in   32 Degrees of Freedom:

Regression model form:
                 c1  c2   c3     c4    c5
P* = P+1 =  e  *V  *Tb  *cosO  *DIA
Reduced Ballistic Limit Equation:
            z2   z3     z4
DIA = z1*V   *Tb  *cosO

The regression model coefficients are:
                        C(1)=     .13686E+01
                        C(2)=    -.11371E+00
                        C(3)=     .22183E+00
                        C(4)=     .57258E+00

P* Critical =    1.9052
The Reduced Ballistic Limit Equation coefficients are:
                        Z(1)=     .28238E+00
                        Z(2)=     .19860E+00
                        Z(3)=    -.38742E+00
```

ANOVA

Source	df	SS	MS	F Value
Regression	3	.47916E+00	.15972E+00	.64285E+01
Residual	32	.79505E+00	.24845E-01	
Total Corrected	35	.12742E+01		

```
Multiple Correlation Coefficient:  (R2)=            .37604E+00
Estimate of Variance:              (SE2)=           .24845E-01

Mean Vector (Xo):
        Velocity          Tb             cosO            Dia
        .51583E+01    .11261E+00     .71897E+00

Confidence Interval:
    Predicted Mean Solution:          P*(Xo)=       1.663
    95% Confidence Interval on P* is:   [  1.577,   1.754]
    The specified value of the t Statistic:         2.029
    Reduced Ballistic Equation Multiplier =         1.098
```

APPENDIX C.3 - FULL REGRESSION OF 385 SHOTS

PROGRAM: mult.for
STEPWISE NON-LINEAR REGRESSION ANALYSIS INPUT FILENAME: full.dat

$$P+1 = c1*V^{c2} *Tb^{c3} *coso^{c4} *DIA^{c5}$$

5 VARIABLES STEPWISE REGRESSED OVER 35 MODELS

=> REGRESSION OVER ALL VARIABLES.

THE LINEAR COEFFICIENTS ARE:

C(1)	C(2)	C(3)	C(4)	C(5)
.6590	.0890	-.0767	.3184	.5138

THE NON-LINEAR COEFFICIENTS ARE:

Z(1)	Z(2)	Z(3)	Z(4)
.9722	-.1733	.1494	-.6196

STATISTICAL RESULTS:

MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
.0347	.4651	110.1251	0	385	.532E-05

=> CONSTANT THICKNESS REGRESSIONS OVER 5 VARIABLES.

THE LINEAR COEFFICIENTS ARE:

R NO.	THK	C(1)	C(2)	C(3)	C(4)
2	.2032	.6853	.1602	.2960	.5969
3	.1600	.7958	.0623	.1419	.4607
4	.1270	.8694	.1462	.5097	.6926
5	.1016	1.2959	-.1694	.4092	.4901
6	.0813	1.3456	-.0897	.5472	.5855

THE NON-LINEAR COEFFICIENTS ARE:

R NO.	THK	Z(1)	Z(2)	Z(3)
2	.2032	.9341	-.2683	-.4959
3	.1600	.7202	-.1353	-.3079
4	.1270	.7228	-.2111	-.7360
5	.1016	.2648	.3457	-.8350
6	.0813	.3020	.1531	-.9346

STATISTICAL RESULTS:

R NO.	MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
2	.0395	.4933	47.2171	0	101	.251E-05
3	.0276	.3335	36.2830	0	149	.108E-05
4	.0217	.7555	61.8090	0	44	-.203E-05
5	.0345	.3381	16.5985	0	69	.235E-06
6	.0202	.6352	15.6716	0	22	.326E-06

=> CONSTANT OBLIQUITY REGRESSIONS OVER 7 VARIABLES.

THE LINEAR COEFFICIENTS ARE:

R NO.	coso	C(1)	C(2)	C(3)	C(4)
7	0.	.6791	-.0934	-.1917	.4849
8	30.	1.8289	-.1953	.2918	.3606
9	45.	.7288	-.0116	-.1429	.6461
10	55.	.0000	.0000	.0000	.0000
11	60.	.5132	.2422	.0881	.7523
12	65.	.9376	.0807	.1429	.6964
13	75.	-.0399	.4950	.1180	.7168

THE NON-LINEAR COEFFICIENTS ARE:

R NO.	coso	Z(1)	Z(2)	Z(3)
7	0.	.9312	.1925	.3954
8	30.	.0375	.5417	-.8090
9	45.	.8778	.0180	.2211
10	55.	.0000	.0000	.0000
11	60.	1.1908	-.3220	-.1171
12	65.	.6566	-.1158	-.2052
13	75.	2.5984	-.6906	-.1646

TATISTICAL RESULTS:

NO.	MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
7	.0323	.1990	14.6609	0	122	.158E-05

8	.0063	.5049	5.6085	0	15	.884E-06
9	.0197	.5247	67.3352	0	126	-.236E-05
10	.0000	.0000	.0000	0	3	.000E+00
11	.0287	.6306	24.7529	0	33	.108E-05
12	.0284	.3791	13.4313	0	48	-.106E-05
13	.0281	.6977	39.2342	0	38	.350E-06

=> CONSTANT OBLIQUITY AND THICKNESS REGRESSIONS OVER35 VARIABLES.

THE LINEAR COEFFICIENTS ARE:

R NO.	cosO	THK	C(1)	C(2)	C(3)
14	0.	.2032	.9251	-.1238	.2432
15	0.	.1600	1.0264	-.0682	.5629
16	0.	.1016	1.5193	-.3631	.3819
17	0.	.0813	1.5771	-.2595	.5344
18	30.	.1600	1.4066	-.2610	.3410
19	30.	.1016	.0000	.0000	.0000
20	45.	.2032	.9360	.0218	.7638
21	45.	.1600	.7733	.1099	.5494
22	45.	.1270	1.1005	.0156	.9833
23	45.	.1016	1.2716	-.1324	.6599
24	45.	.0813	1.2210	-.2002	.2383
25	55.	.1600	.0000	.0000	.0000
26	60.	.2032	.0616	.4272	.7300
27	60.	.1600	.0000	.0000	.0000
28	60.	.1270	.6659	.0313	.7967
29	65.	.2032	.0000	.0000	.0000
30	65.	.1600	.4402	.2158	.7004
31	65.	.1016	.8011	-.1100	.4700
32	65.	.0813	.8361	.0118	.8876
33	75.	.2032	-.2575	.5094	.7274
34	75.	.1600	.0000	.0000	.0000
35	75.	.1270	-.1449	.3082	.4558

THE NON-LINEAR COEFFICIENTS ARE:

R NO.	cosO	THK	Z(1)	Z(2)
14	0.	.2032	.3155	.5090
15	0.	.1600	.5075	.1212
16	0.	.1016	.1012	.9506
17	0.	.0813	.1746	.4856
18	30.	.1600	.1070	.7652
19	30.	.1016	.0000	.0000
20	45.	.2032	.6828	-.0286
21	45.	.1600	.7912	-.2001
22	45.	.1270	.6290	-.0159
23	45.	.1016	.3867	.2007
24	45.	.0813	.0890	.8402
25	55.	.1600	.0000	.0000
26	60.	.2032	2.2226	-.5853
27	60.	.1600	.0000	.0000
28	60.	.1270	.9736	-.0393
29	65.	.2032	.0000	.0000
30	65.	.1600	1.3389	-.3081
31	65.	.1016	.7167	.2339
32	65.	.0813	.8059	-.0133
33	75.	.2032	3.4565	-.7003
34	75.	.1600	.0000	.0000
35	75.	.1270	5.6534	-.6763

STATISTICAL RESULTS:

R NO.	MS_RES	CORR	F STATISTIC	ERR	NUM	SUM R
14	.0460	.0474	1.0445	0	24	.840E-06
15	.0269	.3041	32.7707	0	78	-.184E-05
16	.0501	.1537	2.1801	0	15	.626E-06
17	.0006	.9335	28.0729	0	5	.559E-07
18	.0064	.2735	3.0115	0	11	.654E-06
19	.0000	.0000	.0000	0	4	.000E+00

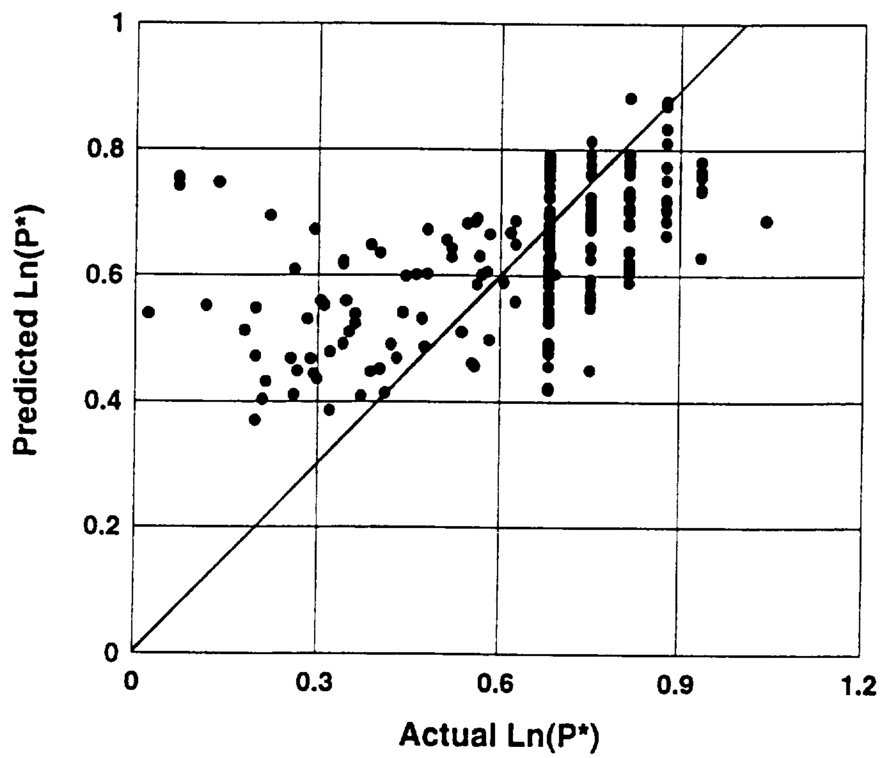
20	.0166	.6247	56.5860	0	37	.753E-06
21	.0181	.4958	30.4878	0	34	.206E-06
22	.0129	.6440	25.3214	0	17	-.523E-06
23	.0286	.5586	34.1693	0	30	-.114E-05
24	.0110	.4713	4.4574	0	8	-.225E-06
25	.0000	.0000	.0000	0	3	.000E+00
26	.0151	.8244	56.3385	0	15	-.506E-06
27	.0000	.0000	.0000	0	3	.000E+00
28	.0243	.6164	19.2818	0	15	-.158E-06
29	.0000	.0000	.0000	0	3	.000E+00
30	.0268	.5973	19.2785	0	16	-.130E-06
31	.0263	.1551	3.1206	0	20	.200E-06
32	.0369	.5291	6.7409	0	9	-.761E-07
33	.0332	.6536	35.8471	0	22	.993E-06
34	.0000	.0000	.0000	0	4	.000E+00
35	.0107	.6544	17.0449	0	12	-.305E-06

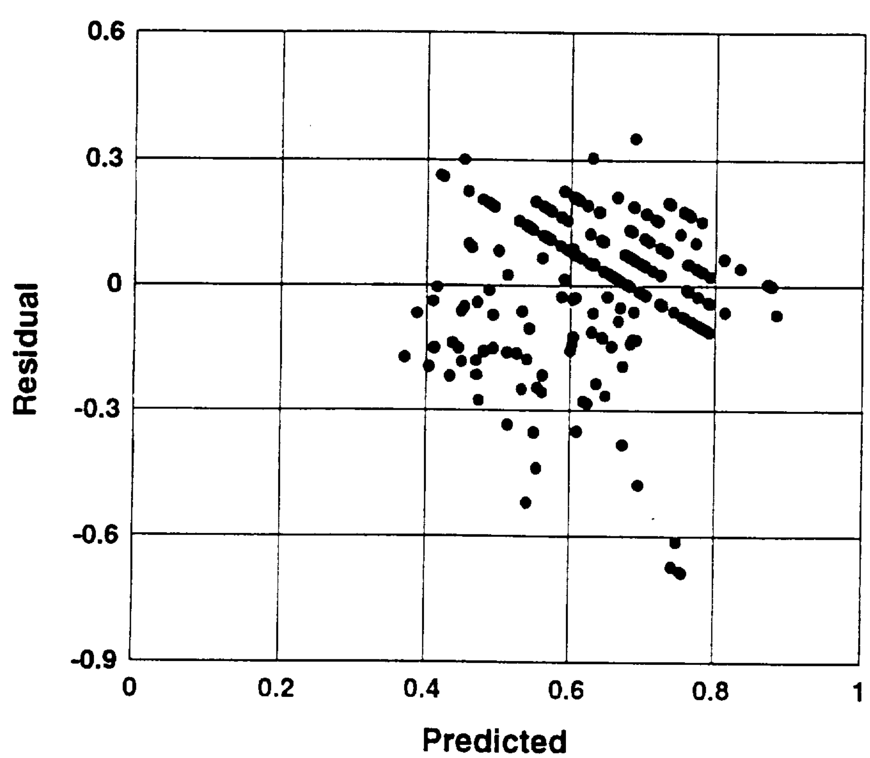
APPENDIX D

RESIDUAL ANALYSIS PLOTS

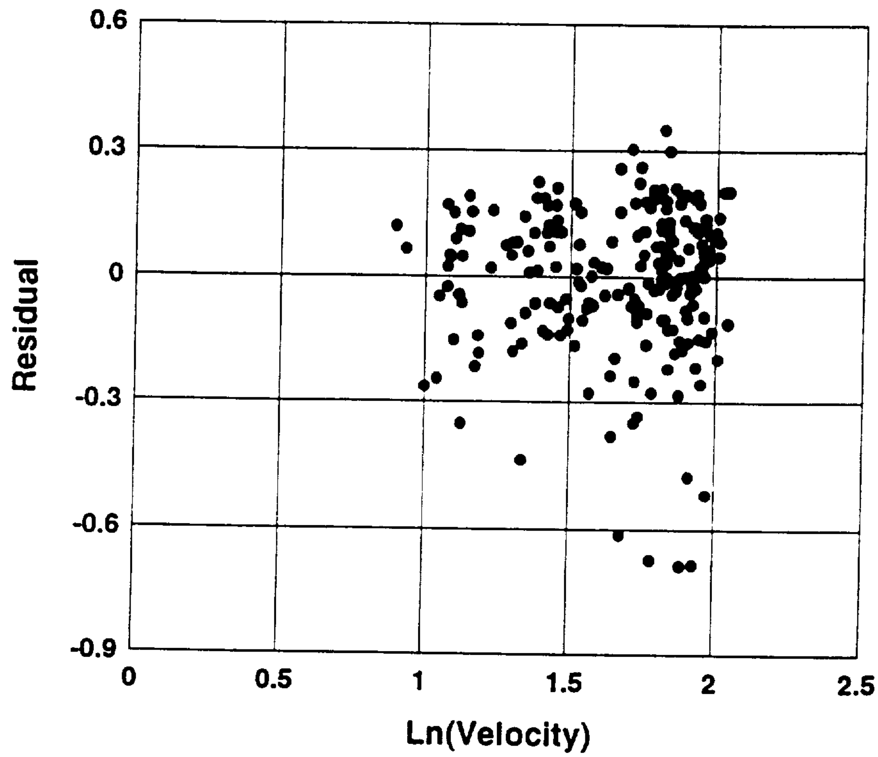
APPENDIX D.1 - RESIDUAL PLOTS OF THE GENERALIZED REGRESSION

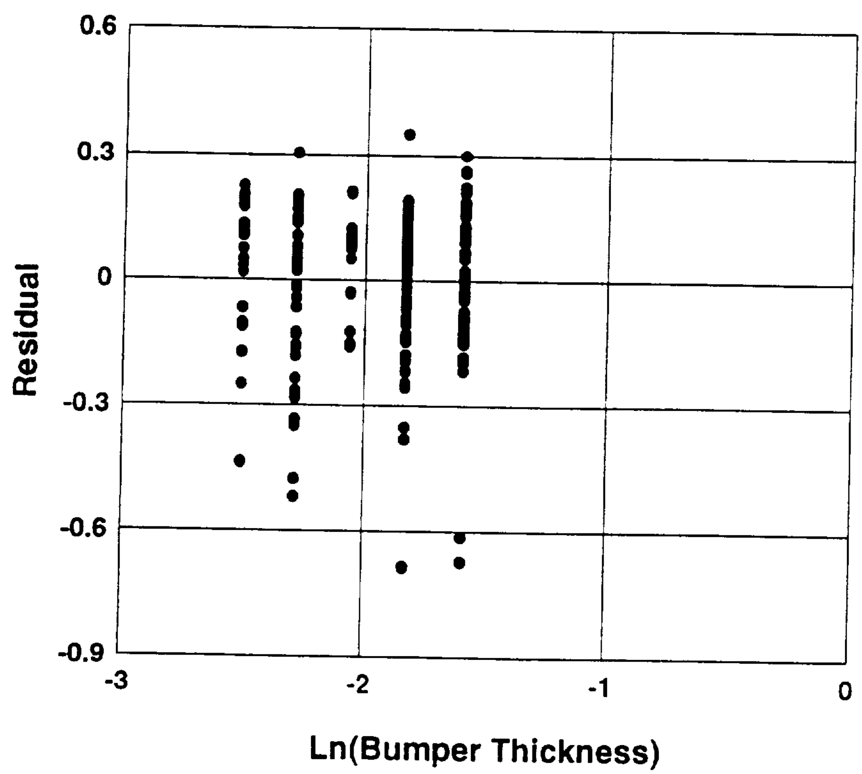
Generalized Regression



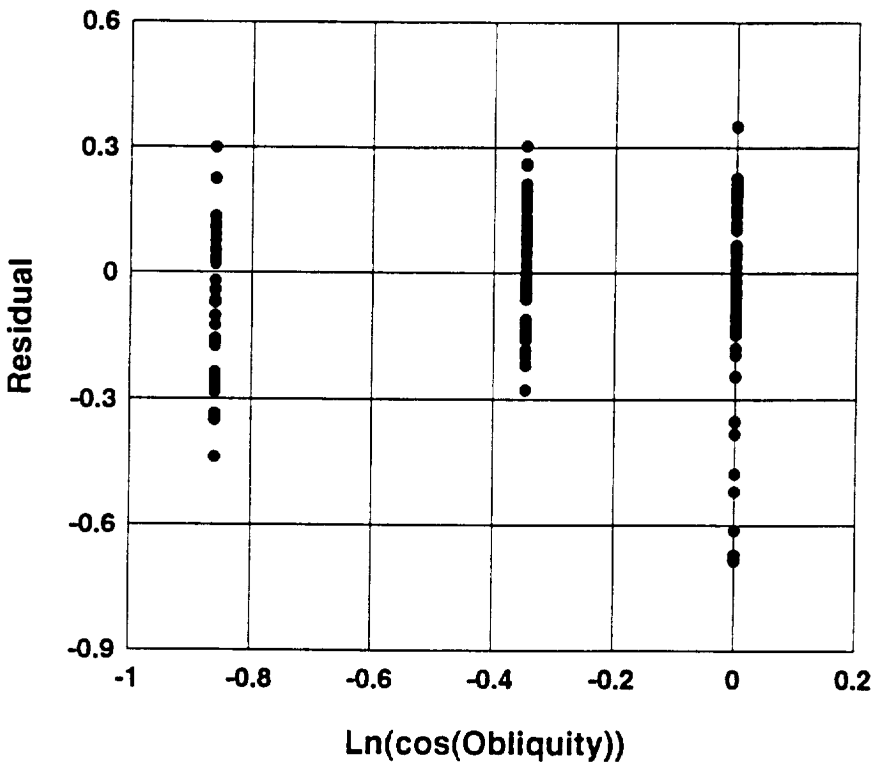


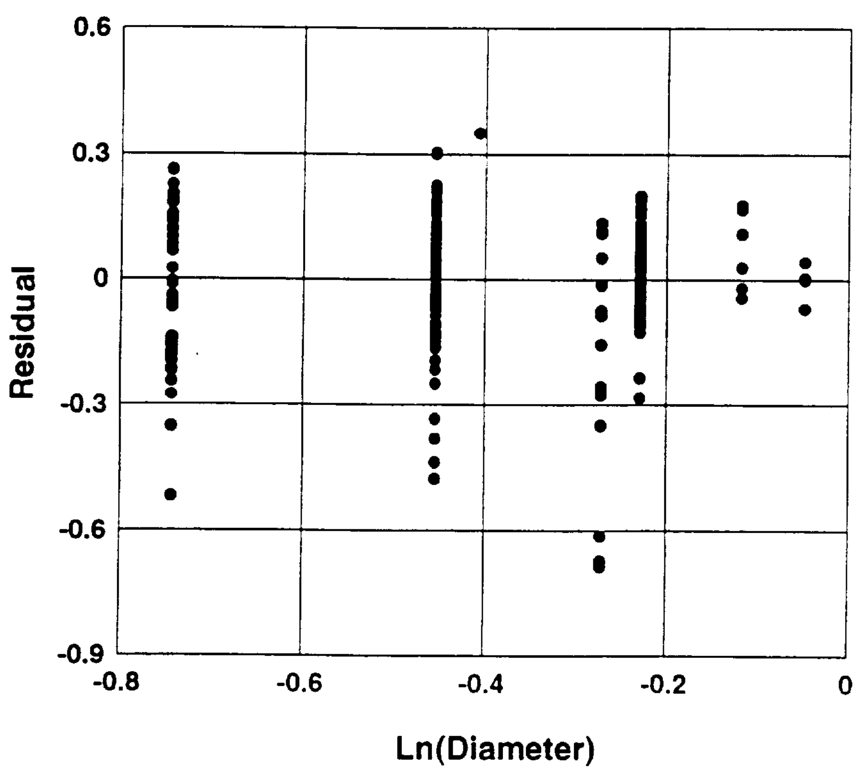
Generalized Regression





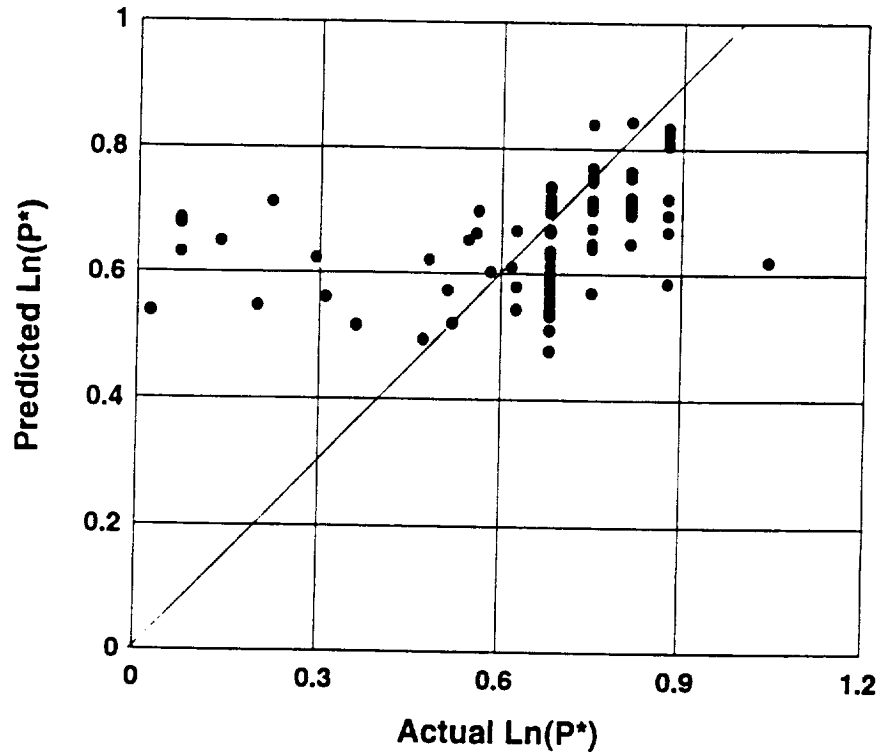
Generalized Regression

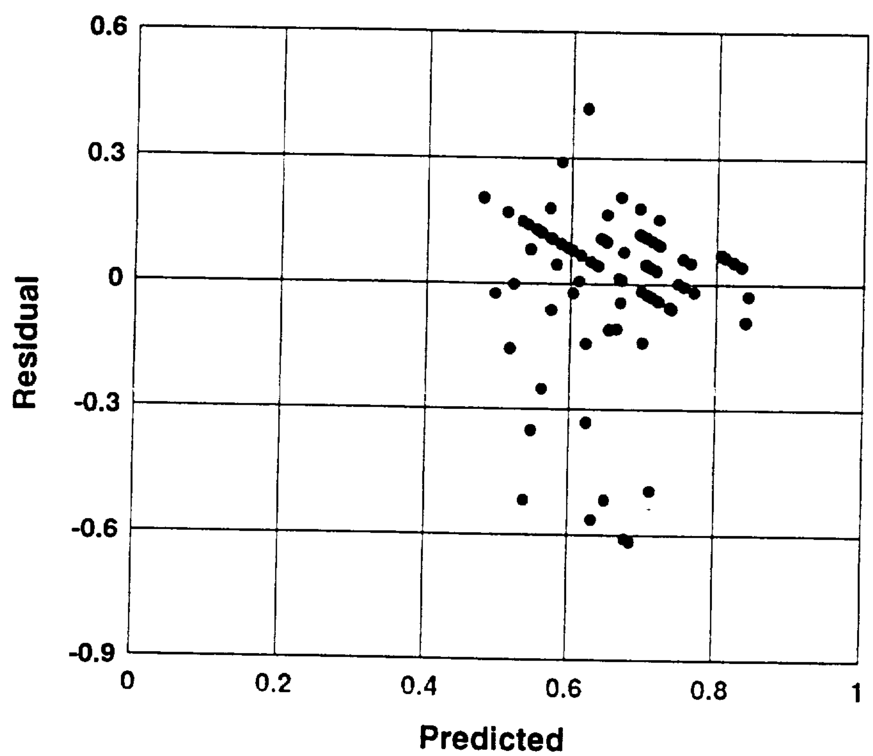




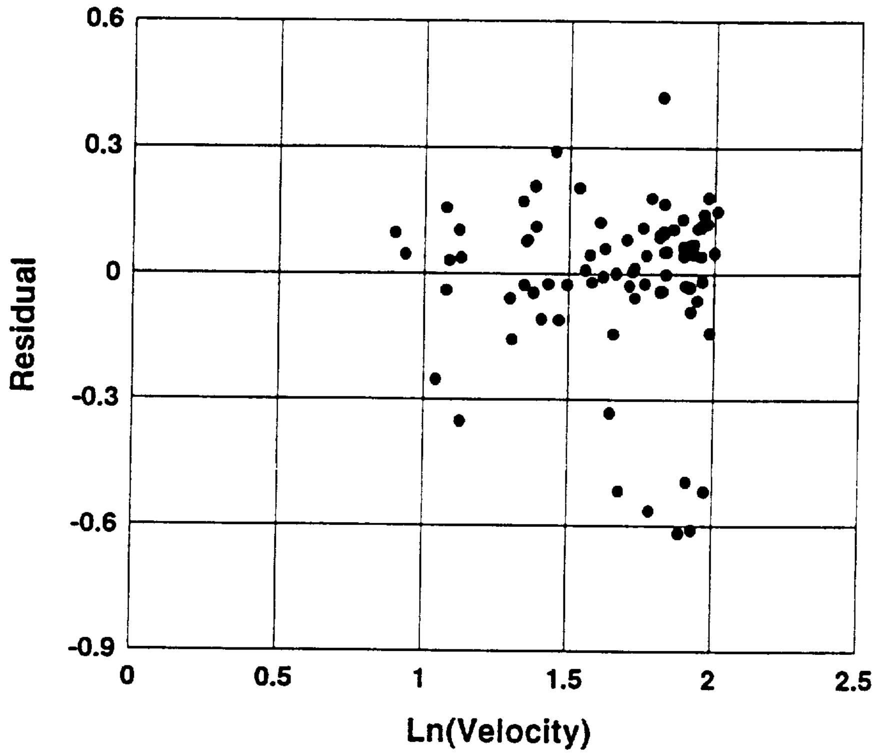
APPENDIX D.2 - RESIDUAL PLOTS OF THE CONSTANT OBLIQUITY 0°

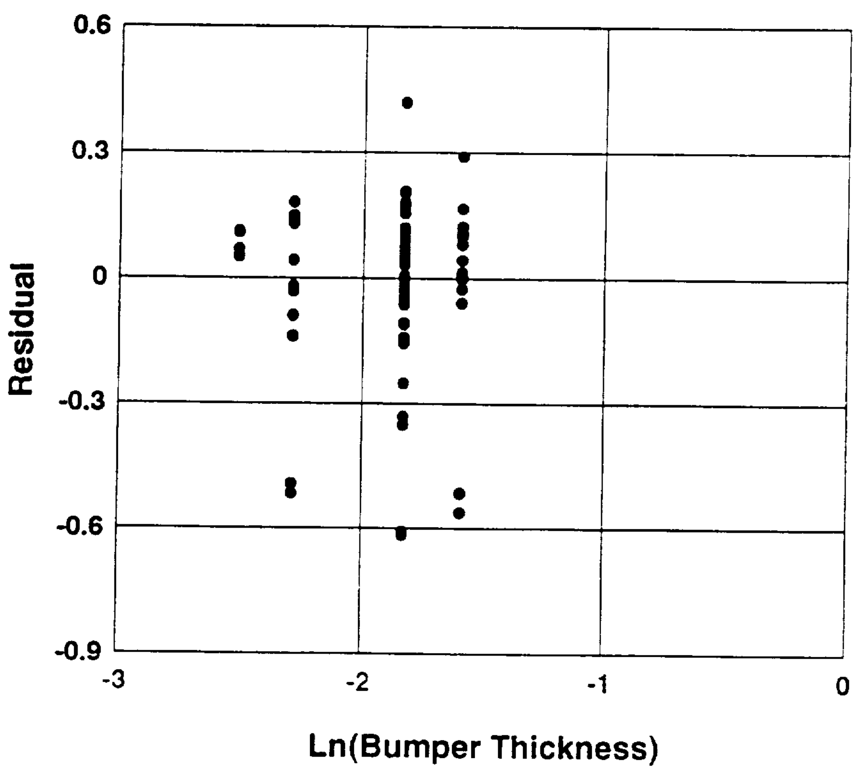
Constant Obliquity 0 Degrees



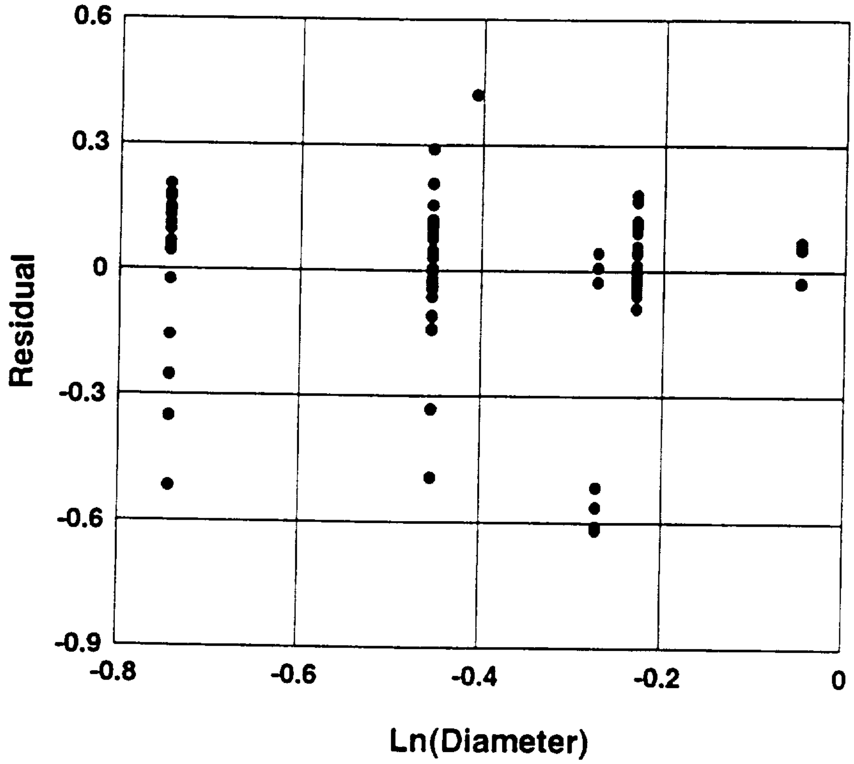


Constant Obliquity 0 Degrees



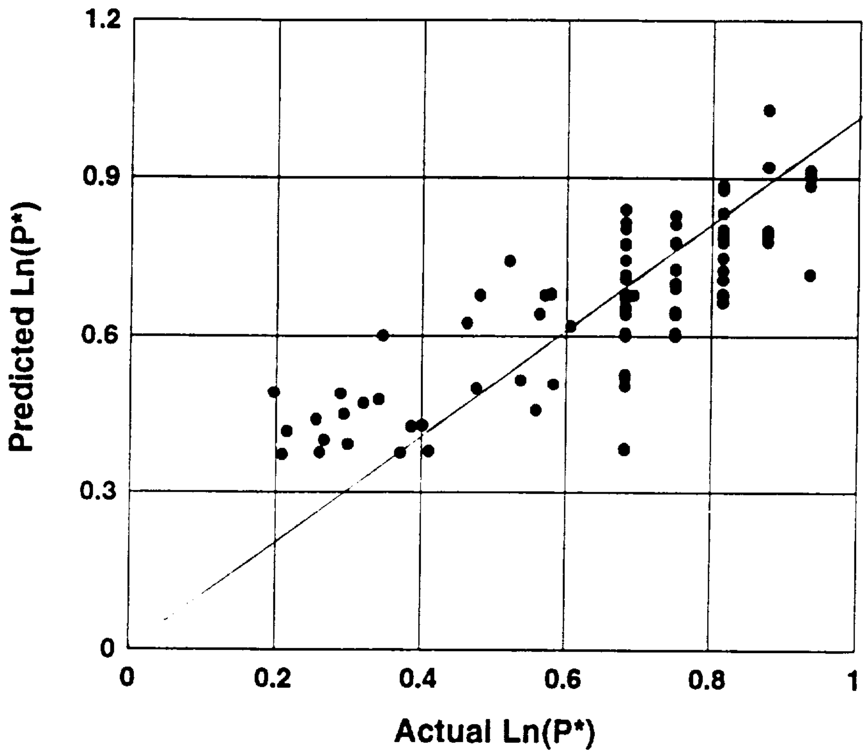


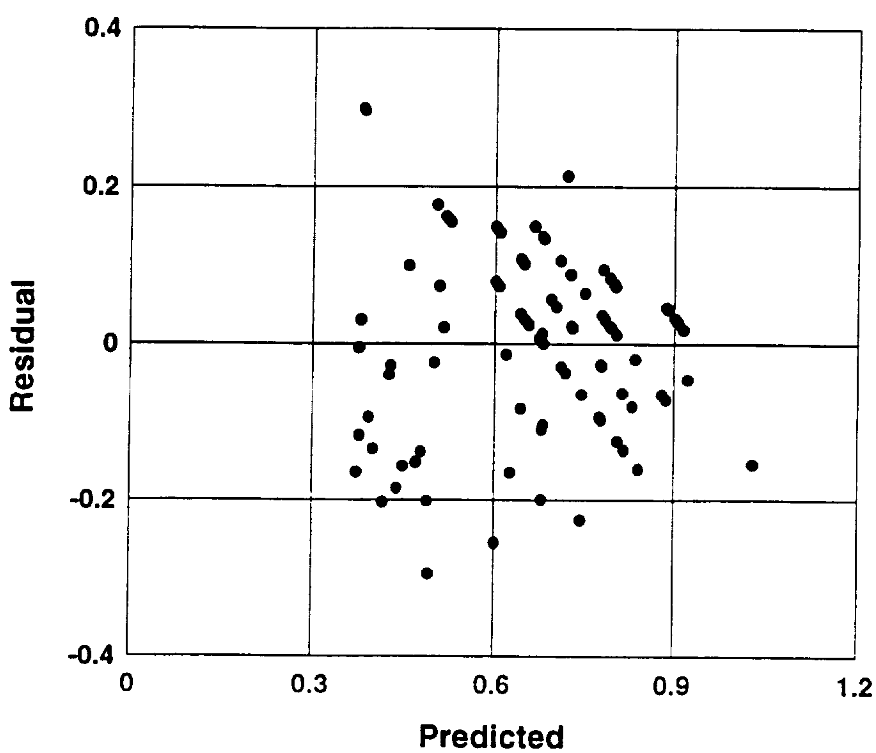
Constant Obliquity 0 Degrees



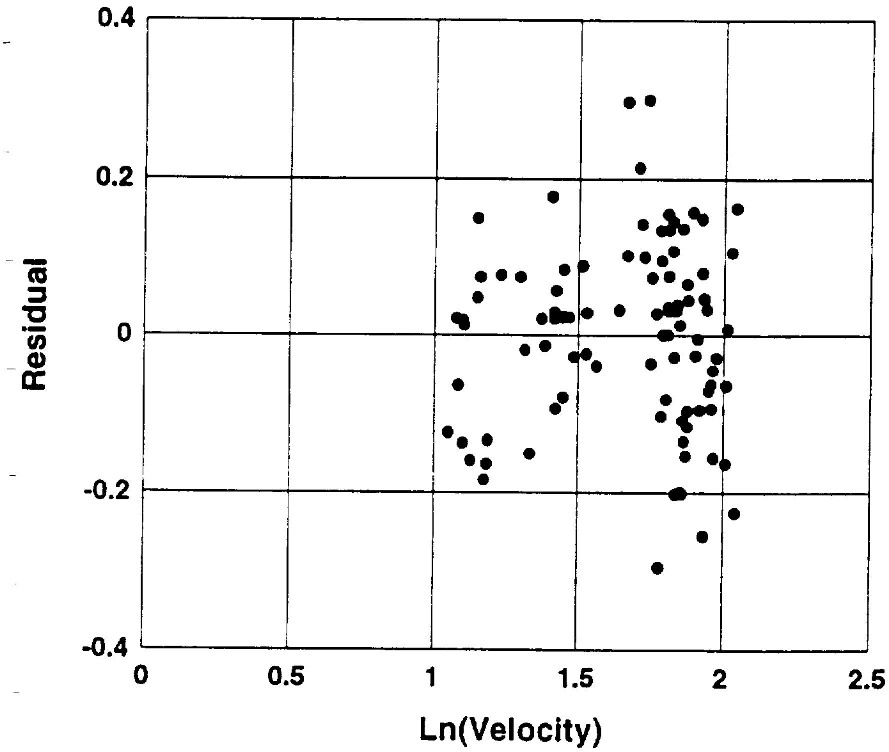
APPENDIX D.3 - RESIDUAL PLOTS OF THE CONSTANT OBLIQUITY 45°

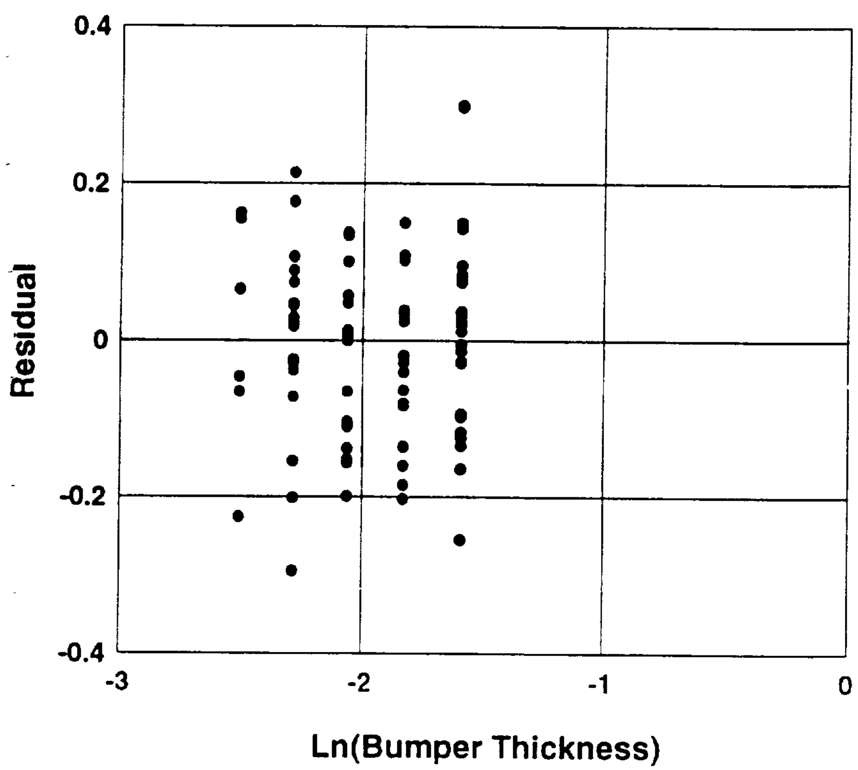
Constant Obliquity 45 Degrees



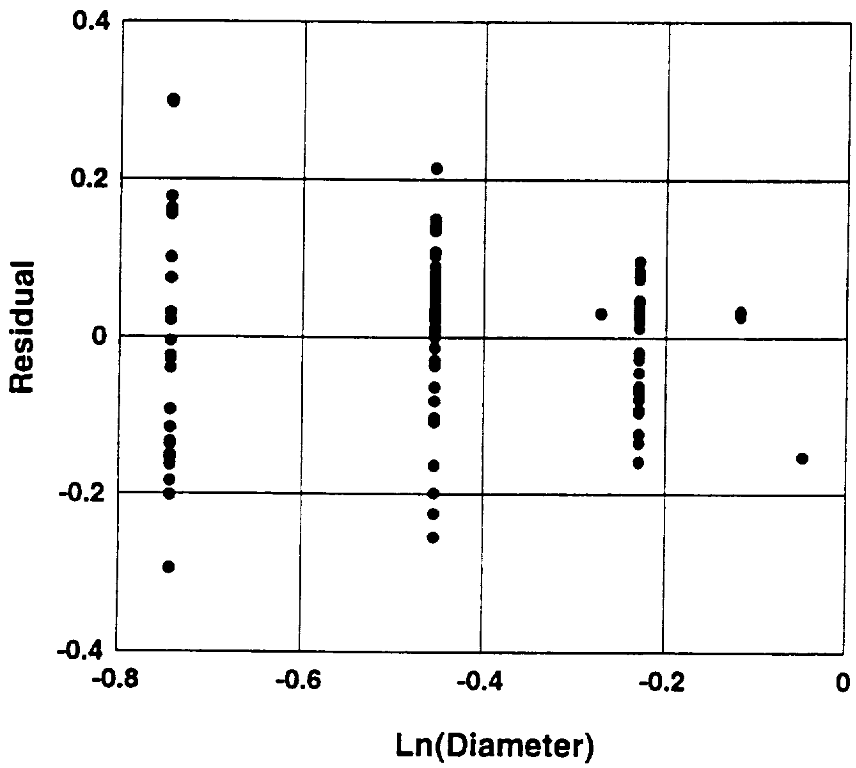


Constant Obliquity 45 Degrees



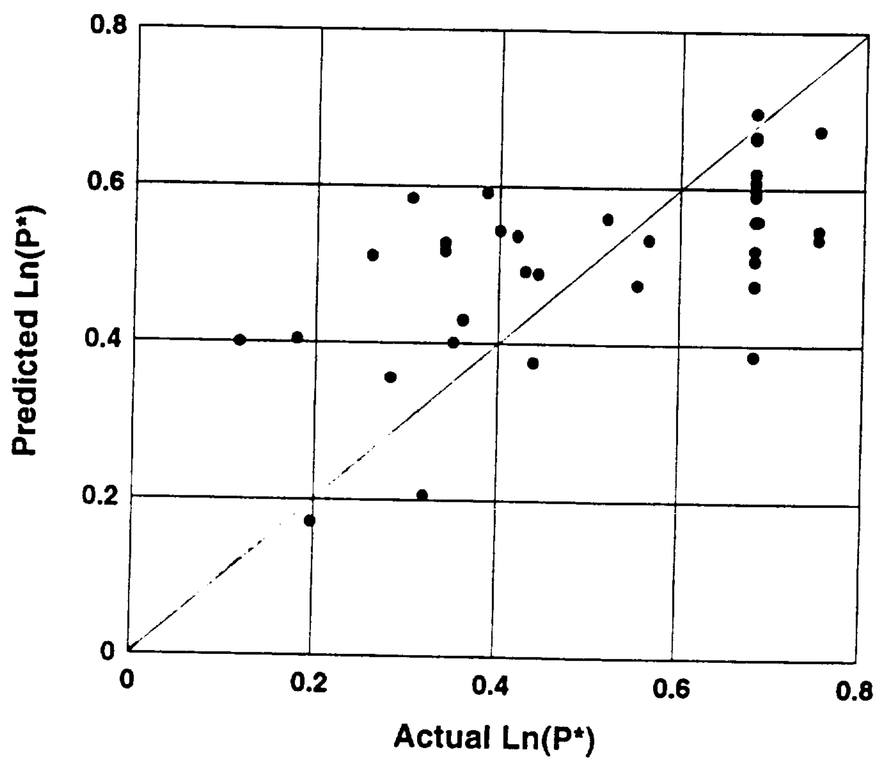


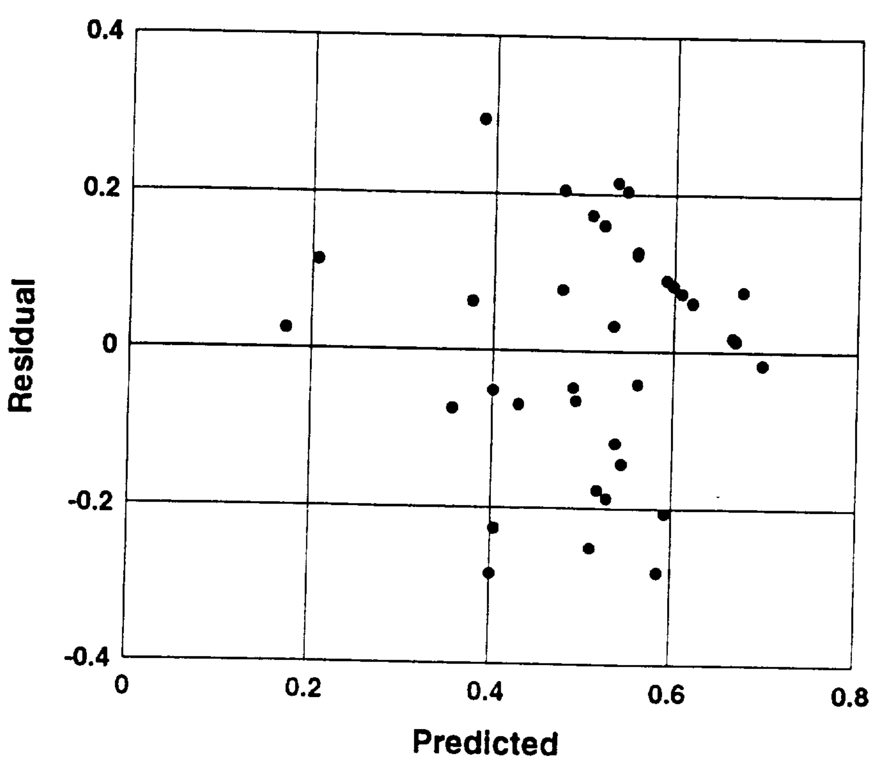
Constant Obliquity 45 Degrees



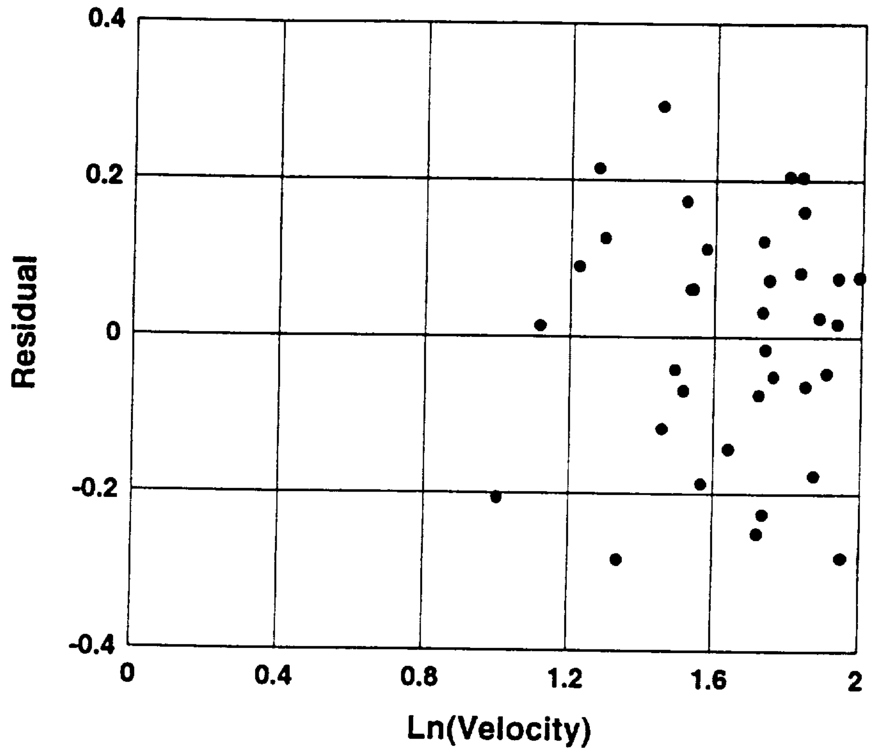
APPENDIX D.4 - RESIDUAL PLOTS OF THE CONSTANT OBLIQUITY 65°

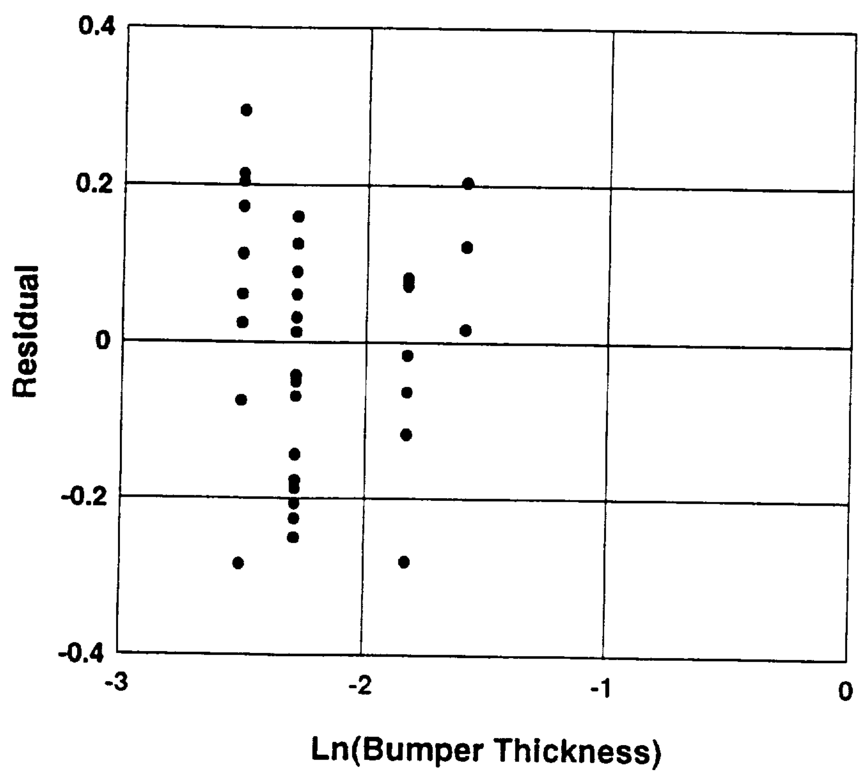
Constant Obliquity 65 Degrees



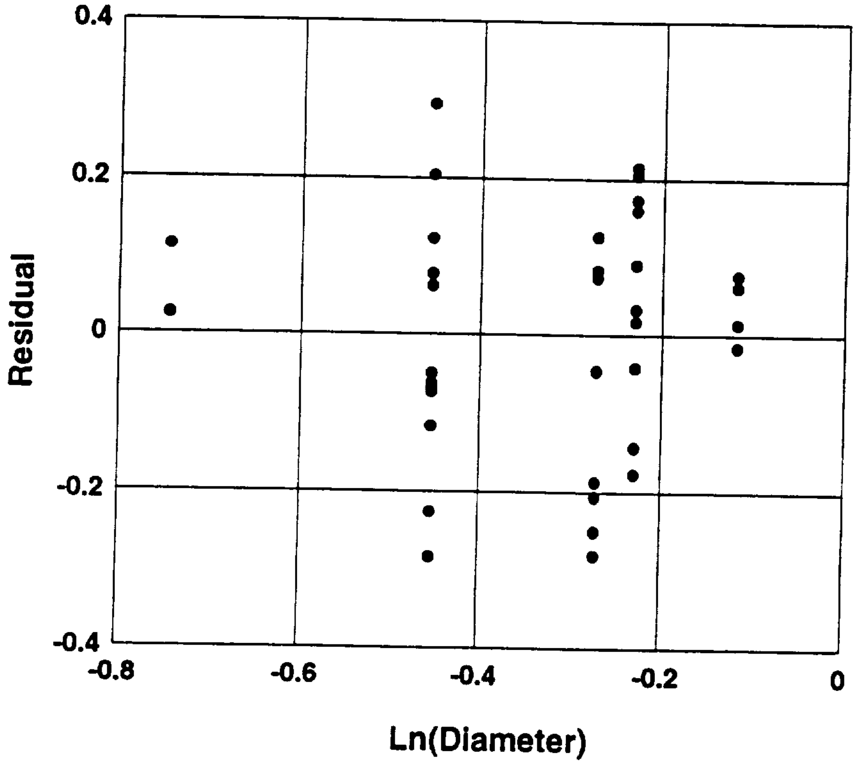


Constant Obliquity 65 Degrees





Constant Obliquity 65 Degrees



APPENDIX E - NON-LINEAR MAPPING OF A MONOMIAL INTO LINEAR SPACE

Confidence Interval:

A set of bounds within which the true mean will lie with an associated probabilty.

Bounds about the vector $\mathbf{X_0}$ are defined by:

$$tol = \pm t\ (v_1 1 - \tfrac{1}{2}\alpha)\ s\ \sqrt{X_0 C X_0}$$

Where:	X_0 (in this case is chosen as the means)
and	C is the inverse of the normal matrix
and	s is the mean square of the residual

Prediction is,

$$P \cdot a^{\pm 1} = e^{C_1}\, v^{C_2}\, t_1^{C_3}\, (\cos\theta)^{C_4}\, d^{C_5}$$

$$\ln P \cdot \pm tol = C_1 + C_2 \ln v + C_3 \ln t_1 + C_4 \ln(\cos\theta) + C_5 \ln \phi$$

let **tol = ln *a*** and right hand side = "**terms**"

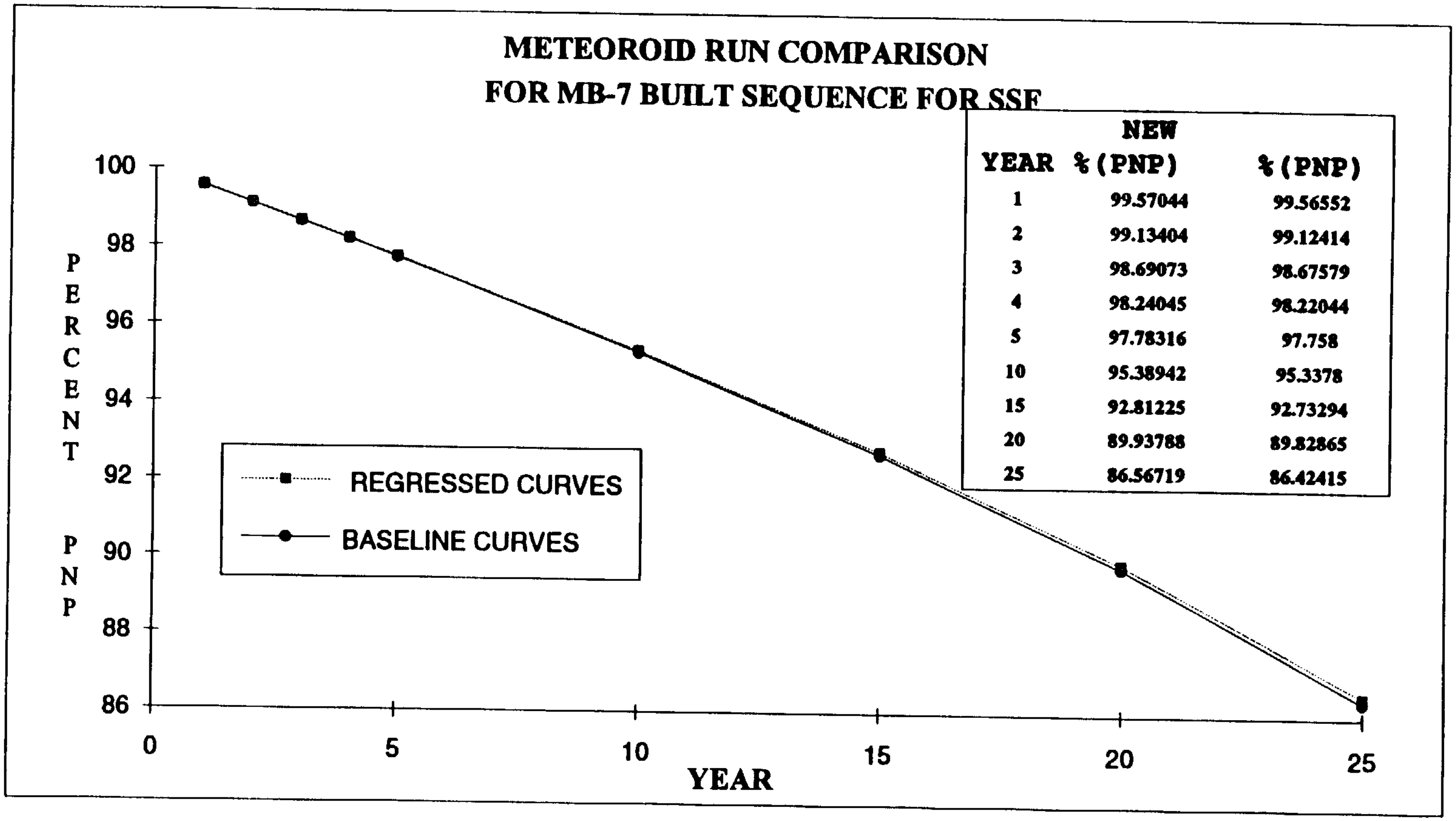
combine left hand side,

$$\ln(P \cdot e^{\pm tol}) = \ln(P \cdot a^{\pm 1}) = terms$$

Back to Non-Linear,

$$P \cdot a^{\pm 1} = e^{C_1}\, v^{C_2}\, t_1^{C_3}\, (\cos\theta)^{C_4}\, d^{C_5}$$

APPENDIX F - PNCF SENSITIVITY STUDY RESULTS



YEAR	NEW % (PNP)	% (PNP)
1	99.57044	99.56552
2	99.13404	99.12414
3	98.69073	98.67579
4	98.24045	98.22044
5	97.78316	97.758
10	95.38942	95.3378
15	92.81225	92.73294
20	89.93788	89.82865
25	86.56719	86.42415